FACTORS AFFECTING ACHIEVEMENT IN HIGH SCHOOL MATHEMATICS:
A SURVEY OF THE LITERATURE

BY

JACOB ISAAC

September 1959

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THESIS
1959(F)
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FACTORS AFFECTING ACHIEVEMENT IN HIGH SCHOOL MATHEMATICS:
A SURVEY OF THE LITERATURE

A Thesis
Presented to
the Faculty of Graduate Studies
University of Alberta

In Partial Fulfillment
of the Requirements for the Degree
Master of Education

by
Jacob Isaac
September 1959

UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES

The undersigned hereby certify that they have read and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "Factors Affecting Achievement in High School Mathematics: A Survey of the Literature", submitted by Jacob Isaac, B.A., in partial fulfilment of the requirements for the degree of Master of Education.

ABSTRACT OF THESIS

Much valuable research pertaining to the factors affecting achievement in mathematics has been conducted in the last two decades. Ready access to this body of research has not been possible for most people, however. In an attempt to place the conclusions of experimental studies investigating these factors in the hands of teachers, the writer of this thesis conducted a survey of the research in this area over the last 20 years.

In order to locate studies concerned with mathematical achievement, the writer examined the Education Index under the headings "Achievement" and "Mathematics," checking all suggestive titles in the listed periodicals. In addition, the tables of contents of seven periodicals dealing with education were canvassed within the time interval concerned.

Nine of the 48 studies located thus were reported on in sufficient detail for a thorough examination. These have been analyzed and evaluated in some detail in Chapter IV. This chapter, therefore, should afford an indication of possible methods and results as well as of difficulties and limitations in research of this type.

The conclusions of all 48 studies have been summarized in Chapter V. All findings have been arranged and reported on according to the origin of the factors concerned; namely, in the community, in the school, in the pupil, in the teacher, and in the subject.

Several important conclusions resulted from the survey. Certain weaknesses in research as it is currently being conducted were brought

to light. The adequate control of variables appeared especially difficult. Further, areas where further research is urgently needed became apparent. In so far as factors were concerned, the effect of many remained somewhat undetermined either because of conflicting results from a number of studies or because of low correlations in a single investigation. Only a few factors, notably the intelligence and the reading ability of the student and the method of the teacher, came forth clearly as significant factors in all studies concerned with them.

It is hoped that the research of the future will be able to overcome the many obstacles facing the investigator and to fill in the gaps in the study of factors affecting achievement in mathematics.

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CHAPTER I

THE PROBLEM

Introduction - The Role of Today's Mathematics Teacher

Mathematics, as a course of instruction in the various types of educational institutions, faces a dilemma today. On the one hand, the progress and security of nations depends upon the acquisition of mathematical knowledge; on the other hand, interest and efficiency appear to be at a low ebb. But not only the welfare of the nation is at stake; the well-being of the individual is dependent in part on a knowledge of mathematics also. As Lancelot Hogben has said, "Mathematics is the language of size, and it is an essential part of the equipment of intelligent citizens to understand this language."[1] Yet the student of today dislikes and shuns mathematics in school.

> Today's pupils, says E. T. S. [Educational Testing Service of Princeton], "just don't like the stuff; they are afraid of it; they don't see any point to it . . . several other studies suggest that mathematics has the dubious honor of being the least popular subject in the curriculum." . . . Nor was this merely a matter of dullness or ability. Of the brightest 30%, four in ten never went beyond elementary arithmetic.[2]

The decrease, percentage-wise, in enrolment in mathematics courses is shown statistically. What of this trend?

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[1] Lancelot Hogben, "Mathematics, the Mirror of Civilization," in Harlow Shapely, editor, A Treasury of Science (New York: Harper and Brothers Publishers, 1958).

[2] "Least Popular Subject," Time, June 18, 1956, p. 56.

> Meanwhile the U. S. Office of Education smugly reports that "percentage enrolment in algebra, geometry, physics and Latin have shown progressive decreases . . . since 1915." Says Bestor [History professor at the University of Illinois]: "It is a peculiarly ostrich-like way of meeting life needs to de-emphasize foreign languages during a period of world war and postwar global tension and to de-emphasize mathematics at precisely the time when the nation's security has come to depend on Einstein's equation $E=mc^2$"[3]

At the centre of this dilemma stands the mathematics teacher. It is his privilege to awaken and sustain interest in mathematics. It is his duty to determine the most efficient of all the teaching methods and, if necessary, to devise new, more productive procedures. It is his responsibility to detect the factors causing disinterest, dislike, failure, and drop-out and to discover means to nullify these factors. The teacher may well be the key to the solution of the dilemma in which mathematics finds itself today.

If this is so, then one of the first steps which must be taken is the determination of all factors which affect achievement in mathematics.

Statement of the Problem

The importance of pupil achievement in mathematics has been indicated. A factorial study as a possible approach to the improvement of this achievement has been intimated. What use has been made of this approach? What, for instance, has research done to answer

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[3] "Nothing Less Than Failure," Time, November 16, 1953, p. 75.

the following questions?

What are the factors affecting achievement in high school mathematics? Why do some students fail to achieve even a relative degree of efficiency, whereas others achieve remarkable proficiency? Certainly the difference in attainment cannot be ascribed to innate ability only. What other factors, then, are operative in determining the degree of success?

Does the community in which the student lives and learns have any bearing on this problem? Is it not possible that social and economic conditions influence the pupil, that customs, practices and beliefs affect his interest and influence the degree of his application to mathematics? And what of the parents with their interests and attitudes?

It is quite logical that the school itself will have some effects on the final outcome of the learning process. There is a difference, conceivably, in the abilities of different types of schools to produce results. Compare the potentiality of an ungraded rural school with that of a large urban school. There may also be some correlation between achievement and the size of the mathematics class. And what of the educational policies of the school?

> Many plans have been devised to provide efficient instruction for pupils of differing abilities. Ability grouping, honors' courses, supervised study, differentiated curriculums and assignments, and individual instruction have been most prominent among the provisions suggested for taking care of both the inferior and the superior student.[4]

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[4] Walter S. Monroe, editor, Encyclopedia of Educational Research (New York: MacMillan Company, 1956), p. 720.

The degree of achievement may be determined in large measure by the student himself - through his ability, his attitude, and his personality. Are these factors? If so, to what extent? And what other factors may reside in the pupil?

It was intimated above that the teacher may be the key to the building up of a sufficient and successful mathematics program. How does the teacher through his personality, his methods, his attitudes and his training influence the results of the instructional program? To what extent do these factors affect the result?

Finally, the question inevitably arises as to whether the subject itself contains factors which will limit the accomplishments of the individual. The curriculum as drawn up, the type of thinking required, the very nature of the subject - may these not determine achievement to some extent?

It is the purpose of this thesis, through a survey of research literature, to try to determine how achievement in mathematics is affected by factors in the community, in the school, in the student, in the teacher, and in the subject.

Delimitation of the Problem

Time

It was felt that the research literature surveyed should be limited to the last two decades, and that for several reasons. First of all, an inclusion of all literature would result in an unmanage-

able number of studies. Secondly, changing conditions in educational institutions and policies as well as in society would tend to decrease the validity or usefulness of earlier research. Finally, progress in research theory and in improvement of techniques would argue for an emphasis upon the latest studies. Therefore, this survey will be limited to the literature of the period from 1939 to 1958.

Grades

Here again, an inclusion of all the grades was found to be impracticable. Once more, unwieldiness was a great factor in deciding to limit the field covered. It was felt, moreover, that the differences in the content of mathematics courses, the approach and methods of instruction, and the readiness of the students in various stages of mathematics would change the set of factors affecting achievement. Thus the field was limited to Grades 7 to 12.

Subject Range

In beginning this survey, it became apparent soon that research studies differed in scope, some being concerned with mathematics as a whole, others treating geometry, algebra, or trigonometry only, still others being limited in their study to very specific fields such as problem-solving in algebra. The investigator decided to include all such studies regardless of scope, believing that each would have some valuable contribution to make toward an answer to the problem under consideration.

Exhaustiveness

It is not the purpose of the writer to give an exhaustive survey of the literature as delimited above. That would require more time and effort than is available. It is his desire, however, that the survey be relatively complete, certainly representative of the total literature available.

In order to fulfil these conditions, the following procedure was resorted to. An examination of the Education Index under the headings "Achievement" and "Mathematics - Junior and Senior High School" was made, suggestive titles being checked in the listed periodicals. The tables of content of the following periodicals were canvassed thoroughly within the time limits stated above:

1. The Mathematics Teacher
2. School Science and Mathematics
3. Journal of Educational Research
4. Review of Educational Research
5. Journal of Educational Psychology
6. Journal of Experimental Education
7. British Journal of Educational Psychology

Whenever literature which was relevant to the problem was found, the bibliography was examined closely for studies not yet located. Thus, while the writer does not contend that an exhaustive exploration of the whole field has been made, he still feels assured that an illustrative portion of the basic research dealing with the factors affecting

mathematical achievement has been examined in the survey.

Implications of the Survey

This survey, it is believed, can be of practical value in the field of education. Of fundamental importance in any attempt to improve achievement in mathematics is a knowledge of the factors affecting such achievement. An awareness of these factors will enable the instructor or administrator to assist, change, modify, or even impede the operation of each factor, as is deemed advisable. In general education, policies, methods, techniques, etc. are dependent on those things which affect achievement; certainly this is the case in mathematics also. This survey will be conducted in a form which will make it useful for information and as source material for such aspects of education as teaching, administration, and teacher education.

According to the writer's knowledge, this survey will be unique in some respects, for, although others have also made surveys of literature dealing with the problem of the identification of factors affecting achievement in mathematics, no survey has been quite as intensive as this one. Monroe has included such a survey in the Encyclopedia of Educational Research, but he has had to be very selective in his choice of literature and very sketchy in his review of the procedures and results. Others have attempted somewhat more elaborate surveys but again have been quite selective of the studies reviewed. These surveys will be indicated in the next chapter under

"Related Literature."

Form of the Survey

After indicating the literature which has been used and defining terms which are involved, a detailed examination of certain key studies will be undertaken. Thereupon will follow a complete listing of the results of all the located studies, arranged according to the five categories of factors: community, school, pupil, teacher, and subject. Specific mention will also be made of conclusions that certain factors exhibited no significant correlation with achievement. It is proposed to continue with a brief indication of some studies which, although not vitally concerned with the problem, are yet significantly related and have important contributions to make to the central problem. An analysis of all investigations, through summaries and tabulations, should then lead to useful conclusions regarding the factors affecting mathematical achievement.

CHAPTER II

LITERATURE

The purpose of this chapter is to give a brief account of the literature which has been used in the present study. This literature can be divided into three different types:

1. related surveys - surveys which are of a similar nature and whose structure and bibliographies were useful to the writer

2. primary studies - experiments which met the requirements of the problem in every respect

3. related studies - experiments which did not meet these requirements but which were closely related to mathematical achievement

Related Surveys

A number of surveys similar in nature to the present survey were very useful to the writer. They provided helpful hints in procedure and organization and indicated some studies pertinent to the problem to be discussed. A brief review of these surveys follows.

Dodes[1] reported on the literature of the past 50 years bearing on mathematics teaching or, as he put it, "the science of teaching mathematics." In general, the experiments examined in his survey met

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[1] I. A. Dodes, "The Science of Teaching Mathematics," The Mathematics Teacher, 46:157-66, March, 1953.

the minimum requirements of valid research, especially with respect to sampling and statistical technique. With just a few exceptions, the experiments which were selected had been performed with mathematics classes. The report was organized according to the following aspects of the science of teaching mathematics: organization of instruction, framework of instruction, motivation of the lesson, specific classroom practices, the homework assignment, and tests as teaching devices.

Gibb[2] reviewed a decade of experimental studies comparing methods of teaching arithmetic. To locate these studies, she examined periodical indexes, reviews of research and bibliographies found in books and articles. Using this technique, she located the source of nine pertinent experiments, only eight of which she could obtain access to at the time. After examining each of these with respect to the nature of the problem, method of teaching, experimental setting, and method of evaluation, she listed the significant conclusions resulting from the survey.

In reviewing the studies which examined problem-solving in arithmetic, H. C. Johnson[3] tried to determine the causes of difficulty in solving arithmetical problems. Under improvement of problem-solving, he found three general types of studies; namely, relative effectiveness of various methods of attacking problems, relative difficulty of

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[2] Glenadine E. Gibb, "A Review of a Decade of Experimental Studies Which Compared Methods of Teaching Arithmetic," Journal of Educational Research, 46:603-608, April, 1953.

[3] H. C. Johnson, "Problem-solving in Arithmetic: A Review of the Literature," Elementary School Journal, 44:396-403, 476-82, March and April, 1944.

various types of problems, and relation of practice exercises to success in problem-solving.

Daniel Harris[4], in reviewing studies analyzing the factors affecting college grades, offered criticisms of research methods, techniques, reporting, etc. His survey was organized according to the following categories of factors: intelligence, personal details, and background, physical data, personality, school, study habits, teaching methods and conditions, incentives and direct motivation, student load, extra-curricular activity and occupational choice.

Walter S. Monroe[5] in the Encyclopedia of Educational Research reviewed the important research in the field of Secondary Mathematics up to 1949. He touched upon factors affecting mathematical achievement as established by research studies. Since he attempted to cover a vast field in very limited space, he was, understandably, very brief and very selective. With but few exceptions, the studies which he reviewed occurred prior to 1939. He concluded with a list of problem areas urgently demanding significant research.

A. H. Turney[6] devoted Chapter II of his study to a survey of the literature bearing on his problem; i.e., factors other than intelli-

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[4] Daniel Harris, "Factors Affecting College Grades: A Review of the Literature: 1930 - 1937," The Psychological Bulletin, 37:125-66, March, 1940.

[5] Walter S. Monroe, editor, Encyclopedia of Educational Research (New York: MacMillan Company, 1952), pp. 717-725.

[6] A. H. Turney, "Survey of the Literature Bearing on the Problem," Factors Other Than Intelligence That Affect Success in High School (Minneapolis: University of Minnesota Press, 1950), pp. 19-42.

gence affecting success in high school. Since his interest lay in this restricted group of factors, no reference was made to studies dealing with the relationship between intelligence and achievement. In his survey of the remaining literature, he resorted to two major divisions of the literature; viz., "(a) a selection of references presenting the status of ability grouping, and (b) all the literature available to the writer bearing upon his specific problem."[7] Although the first division involved only a "selection of references," there still were more studies in this section than in the second division. This would appear to be an indication of the importance attributed by educators to this aspect of education.

Primary Sources

A number of criteria were used in determining whether experiments were to be used in this study as primary sources.

Problem Delimitations

First of all, they must be in the categories indicated above; that is, they must be studies conducted within the last two decades, they must be concerned with factors affecting mathematical achievement, and they must deal with any of the grades from seven to twelve. With respect to the last-mentioned category, one study at the college level was admitted because it seemed to add useful information to

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[7] *Ibid.*, p. 19.

three similar studies within the grade limits set.

Mathematics Classes

Secondly, the experiment must be conducted with mathematics classes. In this respect, a few studies where the experiment covered more than just mathematics classes were accepted also, since the investigator in each instance gave specific conclusions regarding the factors affecting mathematical achievement.

Research Requirements

Thirdly, it was desired and intended that all studies included be valid as research. Hence, wherever possible, they were examined as to their ability to meet the minimal requirements as to form, technique, statistical procedures, and so forth. Due to the paucity of details, the writer could not be sure in a few of the studies as to the validity of the conclusions. Such experiments were included, nevertheless, because they were the only studies which dealt with factors of interest to the investigator.

Mental Abilities

The field of mental abilities required for success in mathematics is a vast one. It is not the writer's intention to cover this area in his survey. A few studies, however, will be introduced in order to indicate the nature and the possibilities within this field.

Related Studies

A group of studies and articles which did not meet the standards listed under "Primary Studies" has been designated as related studies. These writings were included because each had an important contribution to make to the problem under consideration.

Included in this group are a number of studies failing to satisfy the delimitations of the problem in some single feature. Some were conducted prior to the stated time limits; others used different grades; still others considered general high school achievement; a few were concerned with diagnosing achievement or lack of achievement. The reason for inclusion in this thesis varied. In some cases, a factor was considered important by the investigator, yet no study dealing with this factor within the bounds of the problem could be located. Some experiments were very suggestive in nature and conclusions. A few added weighty evidence for conclusions drawn from primary studies which were somewhat weak in method or structure. All had a significant relationship to the problem of the thesis.

A second miscellaneous group classified as related studies included investigations preliminary to future research, indications of new developments in the field of mathematics education, and a few non-research articles which were of a very suggestive nature.

CHAPTER III

DEFINITION OF TERMS

In this chapter, only those terms which are used extensively in the discussion of the problem will be defined. Terms which are confined to one or two studies will be defined as they occur.

Achievement

Central to the thesis problem is the term "achievement." In most studies, this concept was used as the independent variable. It was replaced by "functional competence" in two studies and by "retardation" in a third. All three terms and their relationship to one another will be dealt with in this section.

Two approaches are possible in defining achievement as related to instruction in mathematics. One is the mastery of mathematical skills, facts, techniques; in short, the mechanics of mathematics. The other is the acquisition of general principles, thinking habits, types of reasoning, versatility of attack upon various problems or areas, and so forth. The first group of attainments is easily defined and accurately measured; the second presents serious difficulties regarding definition and measurement. In all probability it is due to these formidable obstacles that most of the studies reviewed have accepted the first approach to a definition of achievement. This being so, the same approach will be used in this thesis.

In most cases achievement will have been defined as a set of

specific skills, facts, procedures, etc., which have been taught in the immediate past and which the student can be expected to have learned. The extent of mastery will have been determined by one or more tests, either self-constructed or standardized. When an investigator has departed from this definition and procedure, special note will be made of the interpretation and approach which he did use.

Functional Competence. The second of the two approaches mentioned above is used in the term "functional competence." This term is used as the independent variable in studies conducted by Alkire[1] and Pitts[2]. Functional competence will have been measured by the Davis Test of Functional Competence in Mathematics. The definition accepted by this test, and therefore by these studies as well, is that of the Commission on Post-War Plans of the National Council of Teachers of Mathematics.

> The Commission of Post-War Plans defines functional competence in mathematics by means of a check list [29 items] which emphasizes the social and mathematical aims of instruction, the need for careful attention to meanings and a more comprehensive program of evaluation, the importance of readiness, the desirability of reconsideration of the importance of sequential instruction in mathematics, the nature of a functional training program for teachers, and the possibilities in the adaptation of multisensory aids to instruction in mathematics.[3]

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[1] G. D. Alkire, "Functional Competence in Mathematics," Journal of Experimental Education, 22:227-36, March, 1954.

[2] Raymond J. Pitts, "Relationship Between Functional Competence in Mathematics and Reading Grade Level, Mental Ability, and Age," Journal of Educational Psychology, 43:486-92, December, 1952.

[3] Walter S. Monroe, Encyclopedia of Educational Research (New York: MacMillan Company, 1926), p. 718.

Retardation. Opposite, in one sense, to achievement is retardation. This concept was used as the independent variable by Hamza[4]. One of the two groups in his study was a class of students specifically retarded in mathematics.

> A case is included amongst the retarded group if the score in any of the attainment tests (arithmetic, algebra, and geometry) is less than $(M - \sigma)$ where M is the mean score for the normal group and σ is their standard deviation.[5]

Intelligence

A second term important because of its frequent appearance is "intelligence." It may be noted at once that, for all practical purposes, the three terms: "mental ability" (in the sense of intelligence), "I. Q.," and "intelligence" are regarded as synonymous. In studies where two or all three of these terms occur, they are used interchangeably.

A formal definition of intelligence will not be attempted here. For such a definition one may turn to any standard text-book in psychology. For purposes of this survey, however, it must be noted that the measurement of intelligence will have been undertaken by any one of a number of standardized I. Q. tests. One must then be aware of the possibility of considerable variations in an individual's I. Q. scores due to differences in the nature, content, and emphases of these

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[4] Mukhtar Hamza, "Retardation in Mathematics Amongst Grammar School Pupils," British Journal of Educational Psychology, 22:189-95, November, 1952.

[5] Ibid., p. 189, footnote.

tests. This is the view expressed by Vernon[6]. It is important, therefore, that conclusions based upon intelligence ratings be considered in the light of the intelligence test used.

Verbal Problems

A number of references will be made to verbal problems. Since slight variations in the definition of this term do occur, it is necessary to specify which variation will be used in this thesis. The present writer's use of "verbal problems" will be limited to "those problems whose solution depends upon equations which the pupil must derive from the facts given in the problem,"[7] the facts being given in word statements.

Unless otherwise stated, the above definition will apply also when reporting on studies conducted by other writers.

Significance

Since many of the studies surveyed are statistical in nature or approach, statistical terms will occur quite frequently. The writer will attempt to use all such terms in the accepted sense as given to them by statistics.

In particular, the noun "significance," together with the corres-

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[6] Phillip E. Vernon, "A New Look at Intelligence Testing," Educational Research, 1:3-12, November, 1958.

[7] E. E. Fulgerson, "Teaching Thought Problems in Ninth Grade Algebra," School Science and Mathematics, 36:393, April, 1936.

ponding adjective and adverb, will be used in a statistical sense. Any other denotation of this word will be strictly avoided. Furthermore, the 1% level of significance will be implied in each case unless a different level is specifically stated.

CHAPTER IV

EXAMINATION OF SELECTED STUDIES

Nine investigations will be discussed in this chapter. Each of these will deal with a different factor or set of factors of mathematical achievement. The nine studies were not intended to be all-inclusive; they were chosen merely because they were the only available studies reported on in sufficient detail to make an analysis possible. The review of these studies should give an indication of possible methods, techniques, and results, as well as of difficulties and limitations in factorial analyses.

The nine studies will be treated individually. Each discussion will present all the information necessary for an understanding of the nature, the procedure, and the results of the experiment. A brief evaluation of the experiment itself will conclude each review.

To facilitate reading, the reports of the nine studies will be kept as parallel as possible. The same sub-topics will be used wherever applicable, and the same order of treatment will be followed whenever practicable.

Enrichment For Superior Students

Albers and Seagoe[1] tried to determine the effect which an enrich-

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[1] Mary E. Albers and May V. Seagoe, "Enrichment for Superior Students in Algebra Classes," Journal of Educational Research, 40:481-95, March, 1947.

ment unit given to superior students would have on their achievement in ninth grade algebra. Enrichment, in this case, was defined as "materials which broaden and deepen the knowledge and content of the regular work."[2]

Construction of Tests and Enrichment Unit

Since no applicable enrichment units could be found, the investigators, assisted by three professors, constructed their own. In doing so, they kept three objectives in mind:

> (1) To develop an understanding of the significance of mathematics in relation to other subjects, the importance of its role in science and engineering, and its significance for the modern world. (2) To introduce the student to certain factual knowledge beyond the usual high school course, including the development of the Hindu-Arabic number system, the improvement of algebraic symbolism and the historical background. (3) Through this enrichment experience to create a lasting interest which might lead to further independent study on the part of the students.[3]

Three parts corresponding to these three objectives constituted the complete enrichment unit. Part I stressed the meaning of mathematics and its significance for our lives; Part II set forth interesting facts about the Hindu-Arabic number system and introduced many symbols used in algebra; Part III depicted the lives of great mathematicians, showing their contribution of revolutionary ideas. The unit as a whole was constructed so as to enable the superior student to study in class time after his regular work had been completed.

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[2] *Ibid.*, p. 482.

[3] *Ibid.*, p. 483.

Since each student proceeded on his own initiative, a minimum of assistance was required from the teacher.

In order to motivate the experimental students to use the enrichment material, self-administering part tests were given upon completion of each part and repeated at the close of the study. An objective test on each enrichment unit served as a measure of differences in improvement between the experimental and the control groups. Finally, the changes in pupil attitudes toward, and interest in, algebra were measured by a questionnaire prepared by the investigators and administered at the beginning and at the end of the experiment.

Selection of Sample

For the purpose of this experiment, only those students having an I. Q. greater than, or equal to, 125 according to the Binet scale were regarded as superior students. The number of such students in each of the two groups, experimental and control, was 32; i.e., 17 boys and 15 girls. These 64 students were located in ten classes in four different high schools.

Four teachers, recommended by the principals of the schools concerned, instructed the ten classes. By means of interviews, the researchers assured themselves of the teachers' full cooperation.

The two groups of superior students meanwhile were shown to be well equated in such significant factors as intelligence quotients, initial algebra achievements, chronological ages, and mean intelli-

gence quotients of the classes in which they were placed. Initial application of the enrichment test revealed a non-significant difference between the means of scores of the two groups.

Procedure

The experiment was conducted in the second semester and included a period of approximately 80 school days with 45 minutes per day devoted to Grade nine Algebra. The experimental students were instructed to keep a record of time spent outside of class time as well as of the class time used for the enrichment material. They had been told of their ability to do the regular work more quickly than the average with the result that they would have spare time available in class, which they would be well advised to use in a self-directed search for many interesting things about mathematics. Most of the students evinced keen interest and curiosity.

Laboratories, which were available in three of the schools, enabled the students to discuss difficult portions of the enrichment unit and to stimulate one another in study and discussion. Study material, with the exception of reference books, could be taken home upon request.

In some cases, teachers set aside a fixed time each week for enrichment work. In any case, all extra time in class could be used for the supplementary material. Thus, while all students continued their regular course of studies, the experimental students had all begun the additional unit not later than three weeks after the begin-

ning of the semester. Only a third, however, managed to complete all of the work set out in the unit. At the close of the experiment, all students were given Form T of the Cooperative Algebra Test, the same enrichment test, and a second interest questionnaire.

Findings

The average time spent on the extra unit by the experimental group was 15% of regular class time plus 56 minutes daily (according to the context) outside of class. This large portion of time was indicative of the group's interest in a project of this type. Further evidence of this interest is available. The experimental group showed a significantly higher increase in mathematics-science interest on the basis of the two interest questionnaires administered to both groups. Unsolicited comments and questions during the course of the experiment gave further evidence of the interest aroused in the experimental students.

The academic results seemed quite favorable also. Firstly, the experimental subjects made relatively the same progress as did the control pupils; this was indicated quite clearly by the Algebra Achievement Tests. Secondly, the results on the two applications of the enrichment test indicated a significant difference in gains in favor of the experimental group.

In so far as student attitude is concerned, the investigators concluded:

> Interest in such enrichment material is sufficiently great to provide necessary motivation for the study. Evidence was strong

> enough to warrant the conclusion that the technique produces a favorable attitude towards Mathematics and Science in general.[4]

Evaluation

The investigators, on the whole, seem to have made a very creditable analysis of the effect of enrichment material upon achievement in algebra. Commendable are the attempts to keep important factors such as I. Q., initial algebra achievement and class mean I. Q., the sizes of the groups, and the ratio of girls to boys constant. Praiseworthy also are the procedures showing the statistical significance of differences in the results obtained.

A few aspects of the experiment might be questioned, however. What, for example, is the effect of the use of students from four different schools with four different teachers? One might argue that four schools would tend to give a better sample than one, but one might also demand that a definite procedure be used in choosing the schools to prevent bias in the resulting population. Would *any* four teachers have obtained the same results, or what would have been the effect of employing teachers who were less cooperative, unenthusiastic, or even unwilling as participators?

It is quite possible also that the situation was more ideal than usual in another respect also. How strong was the motivation to spend time on the enrichment unit? Would the strength of this motivation be maintained over a longer period of time? Was the experimental class

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[4] *Ibid*., p. 483.

aware of the conduct of this experiment? If so, how would this knowledge alter the end result? Would the motivation have been as strong if the class had not been asked to record the time spent on enrichment material?

In spite of these unknown factors, the experiment is of considerable importance and does prove that enrichment material can be of value for superior students.

Improvement of Mathematical Vocabulary

The main purpose of H. C. Johnson's study[5], as stated by the investigator himself, is "to determine whether improvement in specific mathematical vocabulary leads to an improvement in the solution of problems which involve the use of these specific mathematical terms."[6]

Selection of Sample

In performing this experiment, Johnson had recourse to 898 pupils in 28 Grade seven classes of three junior high schools. Of these 898 pupils, 598 were actually used in the experiment, 316 forming the experimental group and 282 constituting the control. Because of the nature of this population, the investigator had two preliminary problems: Did the schools come from the same population?

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[5] Harry C. Johnson, "The Effect of Instruction in Mathematical Vocabulary upon Problem-Solving in Arithmetic," Journal of Educational Research, 35:97-110, October, 1944.

[6] *Ibid.*, p. 97.

Were the control and experimental groups, which had been formed by taking alternating classes from high ability to low ability in each of the participating schools, statistically equivalent at the beginning of the experiment?

The population was found homogeneous with respect to variability (according to the Welch-Nayer test) and to means (according to the analysis of variance test) on each of the following factors:

1. chronological age
2. mental age
3. reading ability
4. knowledge of arithmetic vocabulary studied prior to the beginning of the experiment
5. arithmetic problem-solving and arithmetic vocabulary.

No significant difference was found to exist between the means of the two groups with respect to the first four of the above-mentioned factors as well as to four arithmetical abilities measured by the Analytical Scales of Achievement. The two groups were also found "very much alike" with respect to the nationalities and occupations of the fathers.

Instruction

The experiment extended over a period of 14 weeks. This time was divided into three periods, each preceded and followed by a vocabulary test and by a problem-solving test. The course content of the entire experimental period was confined to five specific

chapters of the text-book currently in use.

In order to ensure unity within the experimental group regarding vocabulary and emphasis, mimeographed material for remedial vocabulary instruction was prepared by the researcher. With a different set of these exercises for each class period of the experiment at hand, the teachers of the experimental classes devoted from five to eight minutes of each period to vocabulary instruction. No extra time was allowed as compensation for this time. The control group proceeded as they would normally have done, learning their vocabulary from the text-book or through class discussions.

Construction of Tests

The tests preceding and following each of the three experimental periods were based upon the material taught during the respective periods. For each word tested for its meaning, there was one problem which was designed to test the application and the understanding of that word. The solution of the problem, however, required a minimum of computational skills, so that what was actually tested was the use of the word itself rather than mathematical mechanics. Each problem was assigned unit value, with no partial credits allowed.

The coefficients of reliability for the tests, using the split-halves method and correcting by the Spearman-Brown formula, were found to vary from 0.77 to 0.91. These reliabilities were considered high enough, "according to accepted practice in educational research," for the kind of group measurements used in the experiment.

In addition to the vocabulary and the problem tests constructed by Johnson, use was made of three standardized tests:

1. Hinmon-Nelson Intelligence Test
2. Gates Reading Test - Ability to Read to Note Details
3. Analytical Scales of Attainment.

The last-mentioned is designed to measure ability in the following phases of arithmetic:

1. understanding of quantitative relationships
2. ability to solve arithmetical problems
3. understanding of mathematical vocabulary
4. ability in the fundamental operations

"A comparable form of this test [Analytical Scales] was administered at the close of the experiment to obtain measures of growth in these general areas."[7]

Findings

The experimental and control groups showed no significant differences in scores in any of the tests preceding the three experimental periods. On each of the post-tests, however, there were statistically significant differences in both vocabulary and problem tests, these differences favoring the experimental group in each case. The lowest gain, still statistically significant for the vocabulary test but insignificant for the problem-solving, for this group occurred in the

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[7] *Ibid.*, p. 103.

third period. The superiority of the experimental students was shown to be consistently significant for all levels of mental ability and to exist even after three months had elapsed. Although the difference between the two groups had narrowed somewhat during these three months after the experiment ceased, it was still statistically significant.

The two sets of data from the comparable forms of the Analytical Scales showed that both groups had made significant gains during the course of the experiment but did not indicate any significant differences between the gains of the groups.

In an attempt to determine the possibility of transfer of training from the vocabulary which had been taught to that which had not specifically been taught, Johnson had added ten "transfer items" to each of the vocabulary tests. The results were far below those required for statistical significance. There had, seemingly, been no transfer to the untaught vocabulary.

Evaluation

Johnson made a very commendable attempt at controlling some extremely significant factors while measuring the vocabulary factor. Through tests and procedures, he ensured that the two groups were representative of the larger population and that they had been satisfactorily equated as regards age, intelligence, reading ability, and pertinent arithmetical ability. The size of his groups tended to add to the reliability and validity of the results. Instruction time was controlled to the extent that it would actually work in favor of

the control group, so that the superiority of the experimental group is especially significant.

Great care appears to have been used in the testing program. A number of the tests used, of course, were accepted standardized tests. In constructing his own vocabulary tests, Johnson was particularly careful to select items involving only elementary computations, since errors due to computational difficulties would have destroyed the validity of these tests. Their reliability had been checked.

Three questionable practices and conclusions could be challenged, however.

First of all, it appears that no attempt was made to keep the ratio of boys to girls the same in the two groups. In the light of general agreement that there are sex differences in achievement in both arithmetic and language, this is a serious criticism. Disproportionate ratios could mean bias in either direction.

Secondly, has the teacher factor been satisfactorily controlled? The investigator reported two checks on this factor, but both fail to be completely satisfactory. He maintained that

> A study in gains in vocabulary and problem-solving made by individual classes revealed that in practically every instance where experimental and control classes were taught by the same teacher, the greater gains were achieved by the experimental class.[8]

Could these "greater gains" have been due in part or in whole to the greater enthusiasm of the teachers for the experimental method? What

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[8] Ibid., p. 108.

were the reasons for the exceptions indicated by the "practically every instance"? He maintained further:

> The efficacy of the remedial exercises (vocabulary instruction) employed in this experiment did not appear to be conditioned by the skill and personality of the teacher under whose guidance and direction they were used. Favorable results were achieved by teachers who were either limited in their experience or who had very little previous knowledge of the needs of the children they were teaching.[9]

It will be noticed that the writer cautiously said "did not appear to be conditioned," for the uncontrolled teacher factors may have more than compensated for limitation in experience and knowledge of pupils' needs.

Thirdly, why should there be such a sharp decrease in the differences between the two groups during the third period? The two explanations advanced by the investigator; viz., reduction of time for remedial instruction and fuller explanation of mathematical terms in the text-book, are not challenged. But may not a third factor be operative as well? Could the interest of the experimental group in this procedure have been waning to the extent that it would affect the results materially?

With these three reservations in mind, we would still have to conclude that Johnson presents preponderable evidence for the positive effect of vocabulary instruction upon mathematical achievement.

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[9] Ibid., p. 108.

Factors Associated With Achievement

Through separate analyses, Schunert[10] investigated the contribution of 34 factors to achievement in geometry and in algebra. These were placed into four categories: pupil, school, teacher, and teaching methods.

Selection of Sample

Having classified Minnesota's 522 public secondary schools with respect to enrolment size and organizational type, Schunert selected a sample of 100 schools by the method of stratified-proportionate random sampling. The same technique was used in the selection of the subsamples which were composed of classes and which were used in many of the 34 comparisons of mathematical achievement. These subsamples varied in size from eight to twenty classes.

Since there was a loss of 23% due to non-response and a further loss of 14% due to incomplete returns, it was necessary to consider the representativeness of the completed samples. Using data on file at the State Department of Education as representative of the total population, Schunert applied the Chi Square technique to check for significant departures from the parent population. He concluded therefrom that the samples used in the analyses were all representative

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[10] Jim Schunert, "The Association of Mathematical Achievement with Certain Factors Resident in the Teacher, in the Teaching, in the Pupil, and in the School," Journal of Experimental Education, 19:219-38, March, 1951.

of the total population at the 1% level of significance.

Construction of Tests

In order to measure mathematical achievement, the investigator constructed an examination which was administered at the beginning and also at the close of the schoolyear. It was based on generally approved objectives and placed approximately equivalent emphasis on each of the following areas of knowledge:

1. knowledge of mathematical concepts and principles
2. mastery of mathematical skills
3. application of mathematical knowledge and skills to the solution of practical problems

The examination showed reliability coefficients ranging from 0.89 to 0.94 as computed by Hoyt's method[11]. It was judged valid by a committee of experts and by an independent sample of teachers in the field.

Three other measures were used: Otis Quick-scoring Mental Ability Test; a questionnaire eliciting information concerning the age, sex, educational plans, and social background of the pupil; and a 90-item schedule obtaining information regarding the teachers' experience, training, and instructional practices. The schedule was organized so as to ascribe quantitative limits, such as "seldom," "frequently," and "regularly," to qualitative categories.

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[11] Cyril J. Hoyt, "Test Reliability Estimated by Analysis of Variance," Psychometrika, 6:153-160.

Procedure

The study was conducted during the 1947-1948 schoolyear. The mathematical achievement test was administered at the beginning of the year and repeated at the end.

The statistical techniques known as analysis of variance and of covariance were used to determine the contribution of each of the 34 factors toward achievement. Any inequalities shown by the initial administration of the achievement test or by the results on the Otis mental ability test were controlled by the analysis of variance. Other factors, such as school size, school organization, experience and mathematical training of the teachers, were kept constant by using the process of random sampling within strata. The appropriate null hypothesis was tested in each of the 34 comparisons.

In employing the techniques of analysis of variance and covariance, Schunert analyzed the data to show that they satisfied the following basic assumptions:

> 1. The variables under consideration must be approximately normally distributed in the population.
> 2. The classes to be grouped together or "pooled" must be homogeneous with respect to the variance of the dependent variable and, also, with respect to the means of the dependent variable.
> 3. The "within groups" regressions of the dependent variable upon the independent variables in the pooled groups under study must be homogeneous.[12]

Findings

The factors found to be significantly associated with mathe-

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[12] Jim Schunert, *op. cit.*, p. 223-24.

matical achievement by categories were as follows:

A. Pupil factors --

1. Boys were superior to girls in geometry.

B. School and class factors --

1. Schools enrolling 100 to 500 pupils showed superior class achievements in algebra.
2. Geometry classes of four-year schools achieved more than those of three-year and six-year organizations.
3. Algebra classes enrolling from 20 to 30 pupils excelled all others.

C. Teacher factors --

1. Algebra classes taught by teachers having more than eight years of experience were superior to those taught by teachers of less experience.
2. Classes of teachers who were graduates of state universities or private colleges excelled those of teachers graduating from teachers' colleges.

D. Teaching factors --

The following factors showed positive significant relationships to achievement:

1. use of regular differentiated assignments in algebra
2. use of regular life applications in algebra
3. review of subject matter more than once a month in algebra
4. failing rate in algebra, not failing more than 2% being better than failing at least 10%

5. supervised study for 20 to 30 minutes per day in algebra
6. testing of geometry classes more than once each week

Mention should also be made of those factors not found to be associated significantly with achievement. In algebra these were sex, type of elementary schooling, type of secondary school attended, amount of assigned homework, extent of pupil-leadership, frequency of tests, and amount of teacher's mathematics training. The following showed no significant relationship to achievement in geometry: pupil's plans for the future, class size, teacher's experience, amount of teacher's training in college mathematics, type of collegiate institute from which the teacher graduated, type and amount of assignments, use of applications, use of pupil-leadership, frequency of reviews, annual failure rate, use of supplementary materials, use of a workbook.

Evaluation

The experiment is certainly a most comprehensive and thorough study. Statistically it appears sound. The investigator, when measuring the effect of one factor, has taken great care to control all other variable factors. He has also avoided the interpretation of causation frequently attributed to factors characterized by mere association. His study is all the more convincing because of the size and the representativeness of the population. Schunert also recognized that in any survey there will always be other factors "unascertained and perhaps unascertainable," which have not been

taken into account and which may influence the results in either direction.

All things considered, one should give serious consideration to his concluding advice to teachers:

> With particular application in the average algebra class, pupil learning would be enhanced:
> 1. if assignments would be differentiated.
> 2. if regular study of life application would be employed.
> 3. if reviews would be made more frequent.
> 4. if more supervised study were used.
> 5. if tests were to be used more frequently.
> 6. if the annual failure-rate would be made less severe.[13]

and to school boards, superintendents, and principals:

> 1. Teachers of more than eight years of experience get better results than less experienced teachers.
> 2. The size of a class for optimum achievement is 20 - 30.
> 3. Schools may be too large (enrolment over 500) or too small (less than 100) to provide optimum educational facilities.
> 4. The four-year organization appears to be the best with respect to geometry achievement.
> 5. Graduates of state universities and private colleges were more successful in algebra teaching than were graduates of teacher's colleges.[14]

A Reorganized Curriculum Based Upon New Objectives

William M. Willits[15] experimented with the effect upon achievement of changing the emphasis, organization and content of the Grade nine algebra course. A new curriculum was necessary, he claimed, because of new objectives. The need for changing the objectives of

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[13] Ibid., p. 237.

[14] Ibid., p. 237.

[15] William M. Willits, "New Objectives for Ninth Grade Mathematics: An Exposition and Appraisal," Journal of Experimental Education, 13:31-45, September, 1945.

algebra instruction lay partly in the stifling influence of traditionalism against objectives based on pupil needs, partly in the ever-increasing number and variability of the high school population, and partly in the social and economic changes which have made the need of instruction in mathematics greater at a time when enrolment is decreasing.

The New Objectives

Traditionalism's narrow objective of imparting vocationally or professionally useful facts, skills and procedures was rejected by Willits. In its place he put a broader, more vital objective -- an objective which emphasized the method of scientific enquiry. The new mathematics curriculum

> must offer more than a systematic body of mathematical learnings . . . more than a preparation for more mathematics or for college . . . more than a promise of usefulness. . . . There is lacking a large, unifying concept, non-mathematical in nature, permeating the stream of mathematical concepts, and recognized as educationally good, the values of which the learner himself can recognize, accept and realize to a satisfying degree. That concept is an expanded idea of the problem-solving process.[16]

He maintained that the heart of problem-solving is "the method of scientific enquiry." Therefore, the curriculum at this point must be organized in such a way that it will foster ability in the use of the six steps of the scientific method:

1. Recognize the problem situation.
2. Formulate the problem.

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[16] *Ibid.*, p. 33.

3. Collect the necessary data.
4. Effect the solution.
5. Test the validity of the solution.
6. Generalize the problem situation.

The Course of Study

Three criteria were used in determining the content of the course; viz., its contribution to the analysis of problem data, its ability to meet the future needs of the pupils, and its offering a reasonable preparation for further mathematics courses for the students desiring to continue in mathematics. On the basis of these three criteria, 24 problem situations were selected as the core of the course. An additional ten problems in the nature of enrichment material were supplied later.

Selection of Sample

Two sections from the Grade nine population of Southern Junior High School in Reading, Pennsylvania, were chosen out of the ten available. The first section, consisting of 37 pupils, was used as a control; the second, with 39 pupils, constituted the experimental group. Preliminary matching not being feasible, the two sections were selected on three bases: (1) similarity of curricular interests, (2) likelihood of this course terminating the mathematical studies of a majority of the pupils, and (3) status as a normal group in the school, the pupils of the school having been sectioned into rapid,

normal, and slow "according to general ability as measured in part by teachers' estimates in terms of marks from the preceding grade and in part by intelligence quotient."[17] Since the groups were found dissimilar on the basis of Otis Intelligence Scores and Metropolitan Arithmetic Scores, the investigator decided to use the class with the lesser ability as the experimental group.

The two sections were taught by two teachers, Willits himself instructing the experimental class in the newly-devised course of studies. The control group was given the regular course of instruction by another teacher.

Procedure

The average time spent per problem was approximately two weeks. Since time was considered secondary to the adequate treatment of each problem, there was considerable variation in the time consumed by the individual problem. The range was one day to five weeks.

No text-book was provided; instead, the students worked from mimeographed sheets. This material, as already noted, focussed attention on the process of solving problems. Special emphasis was placed upon the analysis of the situation, the collection and analysis of data, and the forming of conclusions and generalizations. In order to realize the new objectives, the method used was largely that of pupil participation.

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[17] *Ibid.*, p. 37.

The actual work of solving problems began with a steering committee of four pupils looking up source material, such as necessary information and definitions. Having done so, they presented this information to the class at the beginning of the problem. Discussion following this presentation led, it was hoped, to a better insight into the problem. Where necessary, drills and tests on new skills were interposed. Each problem ended with a discussion to summarize and check results and with a mimeographed test on that problem designed to detect the student's understanding of the problem, skill in new mathematical learnings, and ability to generalize.

Evaluation of results was sought mainly through two tests. The first, the Columbia Research Bureau Test, was used to measure acquired skills and the ability to analyze and solve traditional verbal problems. The second test was constructed by Willits and was designed to measure "abilities more closely associated with the problem-solving process."[18] The reliability of this test, as indicated by the product-moment correlation coefficient between odd-numbered and even-numbered items "left something to be desired."[19] Validity was sought through the judgment of authorities, only those items being retained which had gained a decided majority of responses giving full or qualified agreement.

Additional information about the success of the experiment was

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[18] *Ibid.*, p. 42.

[19] *Ibid.*, p. 42.

obtained from three further sources: comments by pupils on the value of the experiment, a questionnaire about pupil interest in mathematics before and after the experiment, and changes in pupil plans for the future from a course not involving mathematics to one involving some.

Findings

The experimental group at the end of the study was significantly superior in the ability to analyze and solve traditional verbal problems, whereas the control group was superior in acquired skills. The study indicated, therefore, that either skills or the ability to apply the skills will be sacrificed to some degree if one or the other is emphasized.

On the test constructed to measure "abilities more closely associated with the problem-solving process," the experimental group showed a significantly greater mean gain between the pre-test and the final test. According to Willits, this indicated that an emphasis upon understanding the situation beyond its "pure skill aspects" would result in more meaningful learning and in a greater ability to analyze material of a non-mathematical nature than was possible under the traditional method of teaching. This type of mathematics course did not need to be a drawback, therefore, to those students desiring to continue with this subject.

Pupil comments indicated that the experimenter had succeeded in increasing interest in the study of mathematics by emphasizing values while teaching.

Evaluation

There is no doubt that there is a great need for an appraisal of the objectives of high school mathematics courses. The setting forth of new objectives for Grade nine algebra by Willits immediately raises two questions, however. Question #1: What is the value of these objectives? Are they superior to the objectives currently held? Are better objectives possible? Question #2: How can these objectives be realized most efficiently?

The experiment gives a partial answer only to Question #1. A comparison between traditional objectives and the set of new objectives is made on the basis of the two tests mentioned earlier, the purpose being to determine whether the traditional objectives have or have not been realized in addition to the new ones through the approach used with the experimental class.

Question #2 is answered only in the sense that one approach for the realization of the new objectives is proposed and evaluated. This approach consists essentially of a new curriculum set up by the investigator and of a method already in use, i.e., the method of student participation. What portion of the achievement accrued from either one of these factors is not analyzed.

All results and conclusions must take the limitations of the experiment into consideration. Two of these were mentioned by Willits. He admitted that the reliability of the self-devised test was not completely satisfactory. He also admitted that his sample may not have been representative of the total population:

> The appraisal of the proposed objectives was in terms of a single group of pupils in a single school. The pupils concerned were pupils of average general ability in this school; in another school the average pupil may be a quite different individual.[20]

But a third criticism could be levelled at this study. The fact that there were two teachers and that one of these was the investigator could have had significant effects upon the results of such an experiment. There would be the possibility of differences in skill and zeal. There would also be the distinct probability of a more directed (probably also a more resolute) effort toward the specific results of both sets of objectives on the part of the investigator.

In conclusion, then, while this study is strongly suggestive of the superiority of the newly-set objectives, the limitations mentioned would demand that further study be made before their supremacy be accepted.

Intelligence, Sex, and Type of School

Clarence E. Climenhaga[21] investigated the correlation between achievement in Grade eight arithmetic and three factors: intelligence, sex, and type of school. In addition, he tried to determine the adequacy of the course of studies in meeting the needs of Alberta's youth as well as the efficiency with which various phases of arithmetic were being taught.

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[20] *Ibid*., p. 45.

[21] Eugene C. Climenhaga, *A Survey of Arithmetical Achievement of Grade Eight Pupils in Alberta Schools*, M. Ed. thesis, University of Alberta, Edmonton, October, 1955.

Selection of Sample

The schools of Alberta for purposes of this study were categorized as urban, town, graded rural, and ungraded rural according to the following criteria:

> The places considered as cities were those which had received provincial status as recognized by the Department of Municipal Affairs, Province of Alberta. The towns considered were those with a minimum enrolment of two hundred pupils as recorded in the Annual Report of the Department of Education. The graded rural groups were chosen from divisions and counties which were reported as having at least three hundred conveyed pupils within their jurisdiction. The ungraded rural sample was selected from one-room schools in various divisions and counties.[22]

A random sample of classrooms from each type of school was selected by using Lindquist's tables of random numbers. The number of classes to be chosen from each type was determined on the following basis: one classroom per 1000 pupils enrolled in city schools; one classroom per 200 enrolled in town schools; one classroom per 300 pupils conveyed in the graded rural; and ten pupils (rather than a classroom) per 100 pupils enrolled in ungraded rural schools. Using this procedure, Climenhaga obtained a distribution of pupils by types of schools as given in TABLE I on the following page. The disproportionately large number of pupils from urban schools was due to the urban population being larger than the other three populations and to the greater number of pupils per classroom in urban schools. An analysis of the locations of the graded schools and ungraded rural schools selected for the experiment showed that the latter were

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[22] Ibid., p. 6.

TABLE I

DISTRIBUTION OF PUPILS BY SEX AND TYPE OF SCHOOL IN CLIMENHAGA'S SAMPLE

	Urban	Town	Graded Rural	Ungraded Rural	Total
Boys	187	99	105	94	485
Girls	193	123	108	107	531
Total	380	222	213	201	1,016

limited entirely to the central portion of the province and that the former were concentrated heavily in the south-western portion.

Use of Tests

It was felt that the test best suited to the curriculum and emphasis of Alberta's Grade eight arithmetic course was the Iowa Every Pupil Test of Basic Skills in Arithmetic, Advanced, Form O. It is divided into three parts with emphases as follows:

Part I -- vocabulary and fundamental knowledge

Part II -- computational skills in the four fundamental operations

Part III -- problems

The test also yields a total score involving the sum of the three parts.

In order to obtain a measure of intelligence, Climenhaga made use of the Otis Quick-Scoring Mental Ability Test, Beta, Form CM. The purpose of the test, as stated by the author, "is most generally that of finding the degree of brightness of a pupil; that is, obtaining some measure (such as the I. Q.) that indicates the probable rate of progress the pupil will make in school."[23]

Analysis of Data

From the frequency distributions constructed on the basis of the data obtained, the investigator calculated the following statistical measures: means, medians, standard deviations, skewness, and kurtosis, as well as the standard error of each of these measures. The results from these calculations were used in the analysis in the following manner.

The significance of the difference between samples in the basic statistical measures (mean, median, standard deviation, skewness, and kurtosis) was determined from the notion of the critical ratio. This is defined as the ratio between the obtained difference and the standard error of that difference. The probability of obtaining each of the calculated critical ratios was found from Garrett's table for large N.[24]

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[23] A. S. Otis, Manual of Direction for Beta Test, Otis Quick-Scoring Mental Ability Test, p. 8. As found in Climenhaga, op. cit., p. 14.

[24] H. E. Garrett, Statistics in Psychology and Education (New York: Longmans, Green & Co., 1947), p. 190-91.

To ensure the validity of the significance of differences between the means of samples, it was necessary to check the variabilities of the samples. The degree of the variability between samples was determined by the same type of procedure as that outlined above. If the difference in the variabilities of two samples was found to be significant, "the corrected value of 't' [that is, the critical ratio with respect to differences between means], as proposed by the Cochran and Cox approximation method, was considered."[25]

Product-moment coefficients of correlation between intelligence and achievement in arithmetic were calculated in order to establish the degree of relationship between these two factors. The reliability of the coefficient found in this manner was determined by applying Garrett's formula, $t = \frac{r\sqrt{N-2}}{\sqrt{1-r^2}}$, and checking the resulting value of "t" for its level of significance from the table mentioned above.

The measures of skewness and kurtosis with their standard errors were used in determining the extent of departure of the frequency distributions from the normal distribution. The significant negative skewness of all four samples in Part II of the Iowa Test was claimed to be due to the percentage of perfect scores on this part of the test. Only one of the sixteen distributions showed a significant departure from the normal curve in the kurtosis analysis. This departure, Climenhaga concluded, "was a result of some other factor apart from sampling. Extensive practice or drill may have been a

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[25] Eugene C. Climenhaga, op. cit., p. 14.

possible factor."[26]

Findings

The following are conclusions as to the factors affecting achievement.

Intelligence. The coefficients of correlation between mental ability and arithmetical achievement on the three parts of the test ranged from a low of 0.38 to a high of 0.62 in the four samples. The range for the total test was 0.62 to 0.69. In testing the reliabilities of these coefficients against the null hypothesis, it was found that all computed values of "t" were well above the value required for the 1% level of significance. The investigator concluded, therefore, "that intelligence was a definite factor in determining arithmetical achievement."[27]

A curious result was obtained when comparing the correlation coefficients obtained from the part tests with those obtained from the whole test:

> The highest correlation coefficients were obtained from the total arithmetic scores. In three of the cases it was higher than all three subtests and equal to the highest subtest in the fourth sample. This would suggest that a child's total achievement in arithmetic is more closely related to intelligence than any phase of arithmetic.[28]

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[26] *Ibid.*, p. 75.

[27] *Ibid.*, p. 52.

[28] Ibid., p. 50.

Sex. Although the girls showed a superiority over the boys (significant only in the graded rural sample) on the mental ability test, the boys gave evidence of general superiority in arithmetic (again mostly non-significant). There was only one exception: the girls were significantly superior at least at the 5% level in all the samples except the urban, where the boys showed a non-significant superiority. In the total group, the girls excelled in computational skills at the 1% level of significance.

Type of school. The urban schools exhibited a significant superiority over the ungraded rural schools and a non-significant superiority over the other two types. Meanwhile the ungraded rural schools were found to be significantly inferior to schools from all other environments.

Aspects of arithmetic. "Of the three subtests the Alberta children appear to be the most efficient in problem solving; their lowest scores being obtained in vocabulary and fundamental knowledge."[29]

Evaluation

Certain aspects of the experiment are highly commendable. Climenhaga was very careful to control such factors as the variability between groups being compared, the skewness and kurtosis of frequency distributions under examination, and the reliability of

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[29] Ibid., p. 95.

all correlation coefficients found.

Although the procedure used for sampling would seem to guarantee random samples, the result is somewhat odd. As indicated already, the ungraded rural samples are confined to central Alberta, and the graded rural are concentrated in the south-western sector of the province, whereas neither of these two types of schools is represented in the south-eastern portion at all. This is most striking even in view of what is maintained by the investigator.

> A careful analysis of the map would support the randomness of the sampling procedures as the number of pupils in ungraded rural schools is greater in Central and Northern Alberta than in Southern Alberta. Furthermore, the number of conveyed pupils is greater in Central and Southern Alberta than in Northern Alberta.[30]

One might reasonably expect, in this writer's opinion, some check on the randomness of the samples to show that some factor has not biased the selection.

The fact that the girls outnumber the boys in each of the samples, and in two cases by a substantial amount, raises several interesting points. Was this predominance of girls representative of Alberta's Grade eight population? In view of the results in the comparisons by sexes on both intelligence and achievement tests, how have these disproportionate ratios affected the results when comparing types of schools? Remembering the general superiority of the boys in arithmetic, how has the boy-girl ratio affected Climenhaga's conclusion that

> The standardization group of the Iowa Every Pupil Test of Basic Arithmetic Skills appears superior to the Alberta children in arithmetic. In only one of the sixteen cases did the Alberta group

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[30] Ibid., p. 9.

> exceed the median of the standardization group. However, this difference was not statistically significant in every case. The town and urban pupils do not appear to be significantly below the test norms. The very poor attainment of the graded rural pupils suggests some cause for concern.[31]

There is no evidence of any attempt to control the ratio of boys to girls. The conclusions, therefore, refer only to the situation as it was found from the samples collected.

The investigator offered five recommendations for future educational policies and testing programs for the province of Alberta.

Solving Verbal Problems in Arithmetic

Using part of the data collected by Climenhaga, Oscar Fadum[32] attempted a diagnosis of the achievements and deficiencies in the solving of arithmetical problems. While a substantial part of the thesis was not concerned directly with factors affecting achievement, the study has been included in this chapter because of the valuable contribution it does make to the thesis problem. It affords an insight into the areas of greatest difficulty in problem-solving and presents a thorough analysis of the contribution of the factors of environment and sex to achievement in solving arithmetical problems.

Selection of Sample

A random subsample of the 1,016 pupils in Climenhaga's experi-

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[31] Ibid., p. 95.

[32] Oscar Fadum, A Diagnosis of the Achievement of a Sample of Alberta Grade Eight Pupils in Solving Arithmetic Problems, M. Ed. thesis, Edmonton, University of Alberta, September, 1956.

ment, such that the proportion of the four types of Alberta schools would be the same, was required. It was obtained by the method of drawing lot-numbered slips of paper from a container. The result was a sample with a composition as given in the table below.

TABLE II

DISTRIBUTION OF PUPILS BY SEX AND TYPE OF SCHOOL IN FADUM'S SAMPLE

	Urban	Town	Graded Rural	Ungraded Rural	Total
Boys	61	51	48	38	188
Girls	84	43	37	38	202
Total	145	94	85	76	400

To establish the validity of the subsample, "a comparison was made between the mean scores on the problem-solving part of the test in the original sample as reported by Climenhaga, with the mean scores on the subsample used in this investigation."[33] The method of comparison was the same as that used by Climenhaga, the critical ratios between the difference of means and the standard error of this difference being checked for significance against the appropriate table. No significant differences were revealed in any of the four

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[33] *Ibid.*, p. 18.

categories of schools, thereby establishing the validity of the subsample.

Analysis of Data

The study is based on Part III (the problem section), of the Iowa Test used by Climenhaga.

In order to establish the degree of achievement on various types of problems, Fadum first resolved Part III of the test into the following problem categories: simple computation, averages, cost, per cent, fractions, use of formulas, approximations, telling time, and interpretation of diagrams.

For each of these categories, the number of correct solutions was compared with the total possible, these comparisons being made by sex as well as by school type. The percentage of correct scores was then used to obtain critical ratios between the sexes and between the various school types. These ratios were checked for the 1% and 5% levels of significance.

In the diagnosis of students' errors, Fadum calculated the ratio (in percentage) of errors made to the number of solutions attempted. Errors were classified as follows: omissions, the N answers (that is, a wrong choice of the N alternative, where N stood for "None of the preceding answers is correct"), computation, reading and vocabulary, use of denominate numbers, processes in problem solving, per cent, fractions, estimating errors, interpreting drawings and diagrams, and formulas.

Findings

There was considerable variation in the achievement of the pupils in the types of problems represented on the Iowa test. They did "their best work in problems requiring little else than computational skill, and their poorest work in problems requiring knowledge of formulas for correct solutions."[34] There didn't appear to be much consistency in the achievement of pupils from the different types of schools in solving various kinds of problems, except that the pupils from ungraded rural schools were significantly inferior to pupils from other environments.

Considerable difference in the frequency of different types of errors was found to exist also.

Differences in achievement between boys and girls were slight and not too consistent, the differences favoring the boys more frequently than the girls. However, "in graded rural schools, girls frequently score better than boys."[35]

In general, Fadum's study confirmed the observations which Climenhaga made on the basis of the whole test and a larger population. An apparent disagreement in conclusions existed on one point only. Climenhaga found that girls of all environments except the urban were significantly superior to boys in Part II of the Iowa test, which measured computational skills; Fadum, on the contrary, found

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[34] Ibid., p. 85.

[35] Ibid., p. 86.

that boys were superior in all areas except the ungraded rural in the problems involving computational skills. The apparent contradiction may be explained by a greater ability on the part of the boys to recognize the type of computation required by verbal problems.

Evaluation

Fadum draws attention to certain limitations of his study due to the construction of the Iowa test. The use of the "N" alternative has two defects for a study of this kind; the pupil may have arrived at the right answer (in the case where "N" is correct) through faulty reasoning; a choice of "N" where "N" is the wrong answer will not reveal the type of faulty reasoning used. A second limitation lies in the fact that the large number of omitted responses on the test was indicative of insufficient time for the students to complete the test, the contention being that the speed factor should be removed in a test used for diagnostic purposes. Due to these limitations, the investigator stated "the percentages shown in the treatment of the data are relative only, and not an accurate statistical representation of pupils' errors."[36]

It should be noted also that in a few of the categories, the number of problems was so small (as low as one) that the sample resulting was actually too small to guarantee a reliable result.

The value of this study lies in its two-fold contribution to the

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[36] Ibid., p. 4.

teaching of Grade eight arithmetic in Alberta. It shows that Climenhaga's conclusions apply, in general, to the area of problem-solving as well as to arithmetic as a whole. It is also very suggestive of the greatest weaknesses in this phase of education in Alberta. Why do pupils do their best work and their poorest work in the areas indicated by this study? Since the answer to this question is basic to improvement in achievement, further research needs to be done to determine the factors causing these differences in attainment.

Effect of Instruction in Reading Ability

After having shown that research indicated that reading ability was one factor accounting for difficulty in solving verbal problems, F. M. Holmes[37] stated his problem as follows: What effect would actual instruction in the interpretative reading of algebraic material have on the ability to solve verbal problems? Which ability group benefits most from such additional instruction? Which sex?

Selection of Sample

The students of four Grade 10 classes at Bedford Road Collegiate Institute in Saskatoon were divided into two equivalent groups on the basis of:

1. mental ability as determined by the Laycock Test of Mental Ability

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[37] F. M. Holmes, An Experimental Comparison of Algebraic Reading and the Solving of Additional Verbal Problems in Tenth Grade Algebra, M. Ed. thesis, University of Saskatchewan, Saskatoon, May, 1940.

2. crude reading scores as found by the Thorndike-McCall Reading Scale
3. an algebra problems test
4. an algebra reading test

The last two tests were constructed by the investigator. A preliminary try-out of the problems test was made on some "average" classes in different schools of Saskatchewan. On the basis of this try-out, certain items were deleted and others substituted, thereby increasing the number of items from 14 to 16. The reading test, designed to cover the reading items used for instruction in the experimental group, tested the ability to interpret problems, to recognize algebraic symbols, to recognize and use algebraic vocabulary, and to read general material of mathematical content.

Any student who did not write all the tests or who was absent for more than one lesson was not considered in the comparison between the two groups. This left 49 students in the experimental group and 50 in the control. Both groups were divided into two classes for instructional purposes.

Procedure

All classes were taught by the investigator. The experimental period covered the first five weeks of the second term. It involved 24 lessons in addition to testing time. Seven of these lessons were devoted to instruction in the types of equations used in the solution of verbal problems. In the remaining 17 lessons, the teacher con-

centrated on developing the ability to solve such problems. There were five lessons per week, each lesson being 35 minutes long.

The material covered was Chapter XII (on verbal problems) of the regular Grade ten Algebra text-book (High School Algebra by J. J. Crawford), supplemented with a few problems of approximately the same difficulty. The students had had previous contact with verbal problems, but these problems had been less difficult and had not involved three unknowns. The classes had just completed a review of the solution of linear simultaneous equations. The types of problems included were the following: business, number and digit, rate or motion, mixture, mensuration, age, work, miscellaneous.

Normal instructional procedures were followed with the control group. This included explanations of terms when called for by students. No homework was assigned in any classes, and the "lessons taught in the classes of each group were as nearly identical as it was possible to make them."[38]

Each of the 24 lessons with the experimental group was begun by a practice exercise with emphasis on the reading and interpretation of problems and the use and meaning of symbols and vocabulary. Approximately 15 minutes was devoted to these reading exercises, with the first eight minutes allotted to the answering of attached questions and the remaining seven to the marking and discussion of these questions. Instruction in reading included

a. Reading to interpret a problem
b. Reading to recognize algebraic symbolism

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[38] Ibid., p. 27.

    (1) Conventions
    (2) Translating English into algebra
    (3) Translating algebra into English
  c. Algebraic vocabulary
  d. General mathematical reading[39]

The algebra reading test was designed to measure abilities in these four phases of reading. Since the reading exercises were on the same type of material as the initial and final algebra reading test, Holmes concluded that "any difference in gains recorded in doing algebra may be attributed to the effect of the practice given in this type of reading.[40]

The twenty-minute period following the introductory practice exercises was used in work similar to that done by the control group. Since the time spent on the regular part of the algebra course was almost halved, the experimental group completed less problems than the control group.

At the close of the experimental period, the same algebra problems test was administered to all the classes again. They also wrote a second algebra reading test, comparable to the one written at the beginning of the experiment.

Analysis of Data

The comparisons involved a calculation of the means obtained by the groups in the initial and in the final tests in both the algebra

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[39] *Ibid.*, p. 6.

[40] *Ibid.*, p. 26.

reading test and the algebra problems test. This enabled the investigator to calculate the mean gains, the difference between gains and the critical ratio. The significance or non-significance of each difference between mean gains was established from tables showing the probability of getting the calculated critical ratio.

Using this procedure, the following groups were compared with respect to problem-solving ability, algebra reading ability and general reading ability, as measured by the algebra problems test, the algebra reading test, and the Thorndike-McCall test respectively:

1. experimental group - control group
2. boys - girls
3. students of high mental ability - students of low mental ability

Findings

Interestingly enough, in the formation of equations for verbal problems as well as in the complete solution of these problems, the control group showed a greater mean gain although this gain was not significant. On the other hand, the greater mean gain of the experimental group in algebraic reading ability was significant. Both groups showed a non-significant decrease in general reading ability. The difference between these losses, favoring the control, was found to be non-significant as well.

No significant differences between the gains made by the two sexes were found to exist. However, non-significantly greater gains

were indicated for the boys in algebraic reading ability under the experimental method and for the girls in problem-solving ability under the control method.

Under both methods, "students of high mental ability show a definite tendency to gain more than those of low mental ability in algebraic [i.e., problem-solving] ability."[41] No reliable difference in gains was found between the mental ability groups in algebra reading ability. It is interesting to note that the low group gained somewhat more than the high group under the control method.

Evaluation

Holmes indicated the following as definite limitations of his investigation. The unit of teaching was small. The types of reading comprehension were not presumed to be all-inclusive. The study was limited to one grade and to one subject within the grade.

The last-mentioned limitation, however, may be a most desirable one under certain circumstances. Furthermore, it may be a necessary condition, depending upon the purpose of the experiment. In this study, this limitation would be a highly desirable restriction.

In his conclusion, Holmes mentioned a further limitation. The time allotted to the experimental group for actual instruction and practice in the regular algebra work was only 20 minutes per day. Possibly better results would have been obtained if an additional ten

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[41] *Ibid.*, p. 46.

minutes were given to both groups.

How equivalent are Holmes' two groups? Basing his use of the equivalent-groups method on McCall's criteria:

> When the purpose of an experiment is to determine the amount of change due directly to an EF [experimental factor] or EF's, the equivalent-groups method is valid:
>
> (1) Where the total net change in the trait or traits in question produced by irrelevant factors is negligible, or where the amount of such change is measured and discounted by the use of a control EF.
>
> (2) Where it is really possible to equate groups.[42]

he states almost blandly, "as the method chosen for the present study seems to satisfy the above criteria, and was most practical under the condition of the study, it was decided to use it."[43] There is no attempt to show how it "seems to satisfy the criteria." The endeavor to secure equivalence (as far as could be determined from the thesis) was confined to equating on the basis of mental ability, crude reading scores, and sex. What of the effect of other factors, such as personality, home background, and social and economic environment upon the equatability of the groups?

Other factors were controlled somewhat more satisfactorily. The total time allotted to both groups appeared to be equal. The same teacher instructed all classes. The size of the groups was relatively large.

The experiment, this writer contends, raises more questions than

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[42] William A. McCall, How To Experiment in Education (New York: The MacMillan Company, 1926), p. 29.

[43] F. M. Holmes, *op. cit.*, p. 27.

it answers. In view of the fact that other research indicates reading ability to be a factor in the ability to solve verbal problems, why did his control group show greater, although insignificant, gains? Holmes suggested by way of explanation that all the studies he found dealt with Grade nine algebra; but why should reading ability be a factor in solving verbal problems in Grade nine and not in Grade ten? Was the lack of significant results due to the non-equivalence of the groups? Or was it due to deficiencies in the reading material or in the instruction thereof? What of the reliability and validity of the reading and the problem tests? Was the time devoted to instruction in vocabulary (17 lessons only) sufficient to produce noticeable results? The answer to these questions would require a similar, but more elaborate, investigation.

Teacher Attitudes

Design of the Study

Morris L. Cogan[44] sought to determine the effects of specific behaviors of the teacher in the classroom upon the productivity of the students. The latter was to be measured in two ways: "(1) the pupil's performance of required school work, and (2) his performance of self-initiated work instigated by classroom experiences."[45] Both

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[44] Morris L. Cogan, "Theory and Design of a Study of Teacher-Pupil Interaction," Harvard Educational Review, 26:315-42, Fall, 1956.

[45] Ibid., p. 322.

of these aspects were determined by the pupils' responses to questions such as, "When this teacher gives a required assignment, how often do you do it?" The responses were checked by reports of each pupil's performance obtained from the teachers. The teachers' ratings tended to corroborate the estimates of the pupils.

Cogan decided to use three variables of teacher behavior: preclusive, inclusive, and conjunctive-disjunctive.

> In sum, the pupils may perceive the teacher as a person who makes them central or peripheral to the classroom decisions. They may feel that their goals, their abilities, their needs, their "feelings," are taken into account or largely ignored. The teacher behaviors tending to keep the pupils central to the objectives of teaching and to the social interactions of the classroom are termed "Inclusive." Those teacher behaviors tending to give the pupils a position on the periphery of these objectives and social interactions are called "Preclusive."
>
> . . . .
>
> Conjunctive behaviors comprise certain interpersonally "neutral" behaviors of the teacher, in contrast to the variables more centrally involved in the interpersonal relations in the classroom.[46]

As in the case of the measurement of student productivity, so also in the measurement of these three variables of teacher behavior, the pupils' perceptions constituted the major body of data. In an attempt to obtain honest and candid responses, the investigator guaranteed the complete anonymity of all answers and assured the pupils that the sole use of the data would be a statistical treatment in an experiment. In a further effort to ensure the validity of responses, Cogan asked the pupils to report on relatively simple, specific behaviors which they would have had sufficient opportunity to observe.

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[46] Ibid., p. 329-30.

Selection of Sample

In deciding to use Grade eight as the level of his experiment, Cogan was determined largely by the fact that an ever-increasing number of pupils drop out in the higher grades. This would result in a more selective population in the high school grades. He also believed that a larger percentage of the drop-outs might be affected adversely by preclusive and conjunctive behaviors than the percentage of students remaining. Thus a choice of Grade eight would have two advantages: a population more representative of the total school population; and maximum impact of teacher behavior upon student productivity.

The researcher chose schools offering departmentalized instruction mainly because of his interest in "isolating the influence of a single teacher among the many with whom secondary pupils customarily work."[47]

The sample consisted of 987 Grade eight pupils in five New England public junior high schools. They were taught by 33 teachers: 18 teachers of English, 11 of arithmetic, and 4 of science.

Findings

Three general trends were noticed when the students' responses on teacher behavior were compared with their reports on the productivity scale. There was a tendency for the individual student to rate two of his teachers with radically different ratings. Negative corre-

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[47] *Ibid.*, p. 338.

lation between preclusive teacher behavior and student productivity was not found to be significant. But there was strong evidence for a positive correlation between productivity and each of the other two types of teacher behavior, the inclusive variable showing the higher correlation.

When the investigator analyzed the average scores of the teachers to establish whether actual differences in traits among teachers were indicated by the scores, the results substantiated the above conclusions. Several important additional results ensued. He found that the significant differences among the three types of teacher behavior existed in the three subject areas as well. When he grouped the teachers according to the communities where they taught, his data showed that a positive correlation existed between the trait of inclusiveness and productivity for the teachers of each community as well as for all teachers.

Evaluation

The writer of this thesis would like to pose two queries which are of significance in an evaluation of Cogan's experiment. One question would affect the validity of the conclusions drawn; the other would limit their validity to a specific portion of the high school population.

Query 1. Are the pupil responses valid?

The validity of the experiment rests heavily on the ability of

Grade eight pupils to rate their teachers correctly. Are such students able to give a valid rating? Cogan maintained that the ratings which he obtained were valid on two grounds.

First, the responses asked for were reports on relatively simple and specific behaviors. Judgment on this point must be deferred because, although the ability of Grade eight pupils to answer questions of this kind if simple enough is readily granted, yet the level of simplicity and specificity was not illustrated in the report and is, therefore, unknown to us. Thus the validity of the responses in this respect rests upon the investigator's judgment of what constitutes behaviors simple and specific enough for his subjects to observe accurately.

Secondly, "a pupil's ratings for each of the two teachers on whom he reported tended to differ sharply. This is taken to mean that the instrument provided items by which the individual pupil could make consistent differentiations between the behaviors of different teachers."[48] It is just as easy, however, to explain this tendency in other ways. It may be evidence of emotional involvement. Or it may be due to a tendency to remember and emphasize experiences of the immediate past. And there may be other factors as well, which would bring about these sharp differences in ratings.

Query 2. How representative of the total secondary school population are Grade eight students?

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[48] Ibid., p. 338.

Cogan stated: "It would appear likely, then, that a sample drawn from the eighth grade might well be more representative of the total population of secondary school age than one drawn from higher grades."[49] This he concluded from the Bureau of Census statistics giving the per cent of students remaining in school at different age levels: 10 - 13 years, 98.5%; 14 - 15 years, 94.8%; 16 - 17 years, 71.3%. Due to the large percentage of drop-outs in the last age group, Grade eight pupils would tend to be "more representative of the total population of secondary school _age_ than one drawn from higher grades," but it would hardly be representative of the total high school population. If, as Cogan suggested, the drop-outs would have a greater tendency to be affected negatively by preclusive and by disjunctive behavior, then the Grade eight pupils will not be representative of the students of the higher grades. In that case, the conclusions must be restricted to the Grade eight population or, at best, to that population represented by the group chosen for the experiment. The population of interest to the educator, of course, is the high school population as it exists in the schools today.

It is contended, therefore, that the conclusions drawn by Cogan are probably valid for the population of junior high school, but not valid for that of the senior high school.

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[49] _Ibid._, p. 338.

Methods Most Effective For Superior Students

The Problem

Everyone connected in any way with education is aware of the existence of students who are capable of greater intellectual and academic attainments than those of the great majority of students. What can be done to provide maximum achievement for this special group without hampering the progress of the majority? What ways and means will serve this purpose best? This problem is becoming ever more acute due to the lowering of high school standards as more and more of the less talented children are allowed to enter the secondary school. Boredom on the part of the superior student could be one of the greatest factors interfering with his achievement.

The approach used by a study conducted by the National Research Association[50] is different from that of all the other studies reported on. The approach is not experimental but descriptive: "The study reported in this bulletin was intended to discover the points of view of high school principals and the actual practices of junior and senior high schools with respect to the education of superior students."[51] Although different in approach, this survey is still of great value to our problem, for it presents the practices adopted by more than 1,000 schools. As such, it would represent the aggregate

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[50] "High School Methods with Superior Students," National Education Association Research Bulletin, 19:153-200, September, 1941.

[51] Ibid., p. 157.

perception and knowledge arising out of the experience and formal learning of leading educators.

Procedure

Information regarding two aspects of the problem was desired. What were the points of view of high school principals regarding the education of superior students? And what practices had actually been adopted by high schools in this respect? In order to obtain this information, three means were used:

> (1) In 1940 a 6-page printed questionnaire was sent to about 5000 high-school principals representing all types and sizes of schools in all the forty-eight states; (2) a follow-up letter was sent to about eighty of the responding principals, inquiring further about their provisions for superior students; and (3) published literature on the nature and education of such students was examined.[52]

The replies used came from 308 junior high schools, 551 senior high schools and 203 combined junior-senior high schools. Thus, a total of 1,062 schools were represented in the report.

It was necessary, of course, to obtain some uniformity in the conception of the "superior student." The Research Division, therefore, gave a definition as well as information regarding the identification, of the superior student.

The superior student was defined to be "one who is distinctly above the average in general mental ability, in a specific aptitude, or in both,"[53] where the average referred to high school students in

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[52] Ibid., p. 157.

[53] Ibid., p. 158.

general. It was contended that there were only three types of special abilities; namely, musical ability, drawing ability, and mechanical ability, and that abilities in other fields (such as subject areas) were a function of general intelligence. Hence, general abilities could be identified by means of any dependable standardized intelligence examination. Special ability "usually manifests itself in exceptional achievement by the time the child reaches junior high school, providing he has had opportunities for developing the ability."[54]

Mentally superior children, it was maintained, revealed their superiority not only in scholastic achievement but also in physical, social, emotional, and moral traits. Examples of superiority in each of these traits were given by the report.

Theories Regarding the Education of Superior Students

All the principals who responded to the questionnaire agreed that some kind of provision should be made for superior students. A substantial majority subscribed to the idea of providing enrichment material but rejected the use of acceleration in the case of students with superior general ability. They also accepted as desirable the study of more courses in a particular field for students showing aptitude in these special fields. Opinions with respect to other procedures tended to vary to a greater or lesser extent.

Writers in the field tended to agree with the principals in giving preference to enrichment material rather than to rapid grade advance-

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[54] Ibid., p. 162.

ment. Many hesitated to support the separation of superior students into separate classes but did believe that such students

> require more of a teacher than do average students . . . The point is simply that many qualities which are desirable for _all_ teachers are needed in an unusually high degree by those assigned to superior groups. Among such qualifications are high general or special ability, a large stock of pertinent information, good health, freedom from jealousy, training in the psychology and appropriate treatment of superior children, and previous teaching experience.[55]

Current Practices

Approximately 90% of all the responding principals indicated that their schools made some kind of provision for superior students. Provision was of two types: those made through the organization and administration of the school, and those made through the curriculum and instruction.

Organization and administration. The most common practices included permission to carry heavier "loads," individualized instruction, and guidance within classes, and requirement of a higher achievement standard.

The following outline indicates what some of the more specific aspects of school practice were:

1. Acceleration
    a. Extra subjects carried
    b. Rapid advancement in regular subjects

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[55] _Ibid._, p. 168.

   - c. Skipping in the higher grades
2. Differentiated curriculums and course offerings
   - a. Elective subjects
   - b. Parallel courses in certain subjects
   - c. Parallel curriculums or course sequences
3. Ability grouping and enrichment classes
   - a. Comprehensive schemes of ability grouping
   - b. Limited plans of ability grouping
   - c. Special enrichment or honors classes
   - d. Honor schools
   - e. Specialized high schools
4. Extra-curriculum provisions
   - a. Homeroom organization and operation
   - b. Clubs and other activities
5. Special selection of teaching personnel
6. Information used in guiding students
7. Other administrative provisions
   - a. Guidance service
   - b. Unusual policy in regard to "extra" subjects
   - c. Supervised correspondence study

Curriculum and Instruction

More than 75% of the schools reported two modifications of instructional practice for superior students; namely, expectation of more reading and requirement of more abstract and critical thinking.

Modifications in six other points were indicated by 50% to 75% of the schools. These were:

1. Larger amount of work to be accomplished during the term
2. Greater difficulty or complexity of lesson assignments
3. More emphasis on the use of creative imagination
4. Longer lesson assignments
5. Greater provision for experience in group leadership
6. More freedom in choosing and carrying on activities[56]

Six specific enrichment practices in the field of mathematics were indicated by the replies. (1) Some schools furnished supplementary material according to the interests of the superior students. (2) Others varied assignments according to the ability of the students, one school providing three degrees of assignments (minimum requirement, average assignment, and maximum assignment) for each grade. (3) Still others adapted teaching methods to the higher intelligence level of superior students. (4) A fourth group of schools offered enriching activities in addition to the regular course of study. (5) One school selected a group of superior students and assigned them to do special work in mathematics for a four-year period under the direction of one teacher. (6) A school in Ohio "has given some superior students an opportunity to broaden their experience by assuming special responsibilities in other mathematics classes as well as their own."[57]

Findings

A number of important conclusions are derived from the study.

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[56] Ibid., p. 183.

[57] Ibid., p. 193.

1. According to psychologists, rapid advancement of superior students must take the mental, physical, and social maturity of the student into account.

2. Well-conceived and well-administered plans of ability grouping or special classes as well as varied offerings of elective subjects afford unquestionable contributions to the education of gifted students. Separate classes will make the enrichment of the curriculum or the faster advancement of the student much easier. It should be noted, however, that placement in separate classes must be based not only on intelligence test scores but also on school marks, achievement test results, and the judgment of teachers and counsellors.

3. Students unusually gifted in a particular field "should have a curriculum more heavily weighted with courses in the field than is the curriculum of the average student."[58]

4. Teacher qualifications must be considered when appointing instructors to groups of superior students.

5. Extra-curricular activities can make valuable contributions toward skills, knowledge, and the development of leadership ability.

Evaluation

Two limitations must be kept in mind when applying the results of this descriptive survey to the answer of the problem of this thesis in so far as it concerns superior students. Both, however,

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[58] *Ibid.*, p. 194.

leave the survey with a distinct contribution toward the improvement of the superior student's achievement in mathematics.

First of all, the study is concerned primarily with achievement in general. The question remains as to what part of the findings would apply fully to achievement in mathematics. Part of the survey, of course, was concerned with mathematics only; this part could be taken, then, as acceptable practice based upon much experience and knowledge. The rest may still be a good indication of what should be done in mathematics since this subject was considered in all decisions.

Secondly, the survey was conducted almost twenty years ago. Since that time, much knowledge gleaned from experience and from experimentation may have substituted better ways and means of assisting the superior student in his achievement. A similar survey at present would be necessary to ascertain whether changes in theory and practice have been brought about and what these changes are. Such a survey would be highly desirable.

As indicated already, the value of a study of this type lies in the fact that it brings to light the general practices of the day -- practices which are based on the knowledge and experience of key men in the field of education.

CHAPTER V

FACTORS OBTAINED FROM THE PRIMARY STUDIES

This chapter will list the significant conclusions regarding various factors which have been dealt with in the primary studies. These factors will be grouped into five main categories: factors resident in the community, in the school, in the pupil, in the teacher, and in the subject.

Factors Resident in the Community

Five investigators sought to determine whether students from urban schools or students from rural schools were the better achievers in mathematics. Climenhaga[1] and Fadum[2], both using data from a large number of Grade eight pupils in Alberta, agreed that students from ungraded rural schools were significantly inferior in arithmetic achievement to students of urban and town schools. Similar results were obtained from another study: Sanders[3] concluded from his experiment with the algebra achievement of college students that those

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[1] Clarence E. Climenhaga, A Survey of Arithmetical Achievement of Grade Eight Pupils in Alberta Schools, M. Ed. thesis, University of Alberta, Edmonton, October, 1955.

[2] Oscar Fadum, A Diagnosis of the Achievement of a Sample of Alberta Grade Eight Pupils in Solving Arithmetic Problems, M. Ed. thesis, University of Alberta, Edmonton, September, 1956.

[3] Wilma B. Sanders, R. T. Osborne, and J. E. Greene, "Intelligence and Academic Performance of College Students of Urban, Rural and Mixed Backgrounds," Journal of Educational Research, 49:185-95, November, 1955.

who had lived all their lives in urban districts were significantly better achievers than those who had lived in rural districts before entering college. Two other researchers, however, obtained different results. Schunert[4], who investigated the effect upon achievement in high school mathematics of having had elementary schooling in rural or in urban areas, found no significant difference in this respect. Alkire[5] concluded that a high school student was significantly more functionally competent in mathematics if he had taken elementary arithmetic in a rural school rather than in an urban school.

In investigating the correlation between economic status and achievement, Alkire[6] inferred from his data that there was a significant difference between the functional competence of students living in a school district whose assessed value placed it in the upper one-fourth of all districts and the competence of students from a district with a value in the lowest one-fourth. In a similar experiment with colored students, Edmiston[7] decided that economic improvement must result in improvement of social status before it will affect school

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[4] Jim Schunert, "The Association of Mathematical Achievement with Certain Factors Resident in the Teacher, in the Teaching, in the Pupil, and in the School," Journal of Experimental Education, 19:219-38, March, 1951.

[5] G. D. Alkire, "Functional Competence in Mathematics," Journal of Experimental Education, 22:227-36, March, 1954.

[6] Ibid.

[7] R. W. Edmiston, and L. C. McBain, "Social and Economic Background Affects School Achievement," School and Society, 61:190-91, March, 1945.

achievement materially.

Hamza[8] conducted an extensive investigation in the schools of England. He identified the following social and environmental factors of retardation in mathematics:

1. frequent change of elementary or secondary school
2. discontinuity at any stage
3. neglect of filling in the gaps somehow, at home or at school
4. the neighbors and friends and the type of their secondary education
5. poor home conditions
6. interference of parents in mathematics homework

This study would merit special attention because of its scope and method except for the fact that its British setting might invalidate the conclusions for American schools.

Factors Resident in the School

Three studies compared the achievement of students from graded schools with that of students from ungraded schools. Both Climenhaga[9] and Fadum[10] agreed that students from ungraded schools were consist-

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[8] Mukhtar Hamza, "Retardation in Mathematics amongst Grammar School Pupils," British Journal of Educational Psychology, 22:189-95, November, 1952.

[9] Clarence E. Climenhaga, op. cit.

[10] Oscar Fadum, op. cit.

ently and significantly poorer than students from graded schools, whether urban, town, or rural. Martens[11], experimenting with Grade eight pupils, showed that the differences in arithmetical reasoning as well as in arithmetical achievement between graded town pupils and one-room rural pupils is highly significant at the level of the experiment. The question as to whether this difference was due to the town-rural or the graded-ungraded factor or both was left unanswered.

Two experiments dealt with the effect of the size of school enrolment upon achievement. Alkire[12] determined that functional competence in mathematics was significantly related to enrolment, students in schools enrolling more than 500 being more competent than those in other schools. Schunert's conclusion[13], which was made on the basis of pupil achievements on self-constructed content examinations, was not in complete accord with Alkire's, due presumably to the difference in emphasis in the achievement factor. Schunert maintained that students in schools enrolling from 100 to 500 pupils were significantly higher than students in other schools.

Schunert experimented with two other factors residing in the school. When he compared the effectiveness of different school organizations, his results indicated that the achievement of geometry classes of a four-year school exceeded that of classes of the three-year and

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[11] Clarence C. Martens, "Educational Achievements of Eighth-Grade Pupils in One-room Rural and Graded Town Schools," The Elementary School Journal, 54:523-25, May, 1954.

[12] G. D. Alkire, op. cit.

[13] Jim Schunert, op. cit.

the six-year institutions. Comparing sizes of classes, he arrived at a class of 20 to 30 pupils as affording optimum achievement. Note, in this respect, what Monroe concluded:

> Conflicting results have come from various efforts to study the effect of class size upon instructional efficiency in mathematics. The source of contradiction seems to be largely in the absence of agreement upon definition of fundamental concepts. What is meant by a large class, a medium class or a small class?[14]

Guiler[15] found that an increase in the length of class periods did not bring about any significant increase or decrease in achievement.

Factors Resident in the Pupil

The most obvious pupil factor to experiment with is intelligence. It was, therefore, the one most frequently investigated. Out of 22 studies which concerned themselves with factors resident in the pupil, exactly one-half considered it in some form.

The results ranged from very high and significant to very low and non-significant correlations between intelligence and achievement. Climenhaga[16], Davis[17], and Stein[18] found a strong, definite relation-

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[14] Walter S. Monroe, Encyclopedia of Educational Research (New York: The MacMillan Company, 1956), p. 723.

[15] W. S. Guiler and Vernon Edwards, "An Experimental Study of Methods of Instruction in Computational Arithmetic," The Elementary School Journal, 43:353-60, February, 1943.

[16] Clarence E. Climenhaga, op. cit.

[17] Robert A. Davis and Marguerite Henrick, "Predicting Accomplishment in Plane Geometry," School Science and Mathematics, 45:403-05, May, 1945.

[18] Harry L. Stein, "Characteristic Differences in Mathematical Traits of Good, Average, and Poor Achievers in Demonstrative Geometry," The Mathematics Teacher, 36:164-68, April, 1943.

ship between the two factors. Somewhat more moderate relationships were indicated by the investigations of Ebert[19], Guiler[20], and Rosilda[21]. In the latter group one would probably find Hamza[22] also. When he compared retardation with intelligence, his results showed that the retarded students were generally of low intelligence. Kellar[23] cautiously said that the correlation between achievement and intelligence was probably significant. The wide range in the coefficients of correlation as found by Challman, a range which extended well below the reliable, led her to conclude that "one would not be justified in assuming that an I. Q. score based on one mental test would always be a reliable criterion for guiding pupils into mathematics courses."[24] Partialling out age and reading grade level, Pitts[25] found that scores on the Davis Test of Functional Competence in Mathe-

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[19] Reuben Ebert, "Generalization Abilities in Mathematics," Journal of Educational Research, 39:671-81, May, 1946.

[20] W. S. Guiler and Vernon Edwards, op. cit.

[21] Sister M. Rosilda, "Is an I. Q. an Index to Algebra Ability?" Journal of Educational Research, 44:391-93, January, 1951.

[22] Mukhtar Hamza, op. cit.

[23] Wylma Rose Kellar, "The Relative Contribution of Certain Factors to Individual Differences in Algebra Problem-solving Ability," Journal of Experimental Education, 7:26-35, September, 1939.

[24] Mildred Challman, "The Retention of Arithmetic in Relation to Achievement in Plane Geometry," The Mathematics Teacher, 39:79, February, 1939.

[25] Raymond J. Pitts, "Relationship Between Functional Competence in Mathematics and Reading Grade Levels, Mental Ability and Age," Journal of Educational Psychology, 43:486-92, December, 1952.

matics were not significantly related to I. Q. scores obtained from the Otis Quick-Scoring Test of Mental Ability. Another study worthy of note at this point has implications in the field of transfer of training: Clarke[26] found that one of the factors affecting the development of reasoning ability through geometry instruction is the mental ability of the student.

This disagreement as to the role of intelligence in achievement supported the conclusion of Carroll: "For the population studied, the bond between mental ability and academic achievement appears to be smaller than is usually assumed,"[27] and of Monroe: "Available evidence leads to the conclusion that achievement in mathematics is not so closely related to general intelligence as it was once thought to be."[28]

A second factor which resides in the pupil and which has been investigated frequently is sex. Of the studies located, six dealt with this factor. All agreed that boys generally are better achievers than girls in mathematics, but in most cases this difference was found to be non-significant. Two researchers did find significant differences: with respect to Alkire's[29] functional competence in mathematics,

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[26] S. C. T. Clarke, Clear Thinking Through Geometry, M. Ed. thesis, University of Alberta, Edmonton, May, 1943.

[27] John B. Carroll, "The Factorial Representation of Mental Ability," Educational and Psychological Measurement, 3:330, 1943.

[28] Walter S. Monroe, op. cit., p. 722.

[29] G. D. Alkire, op. cit.

boys scored significantly higher than girls; Schunert[30] found that the boys' achievements were significantly higher than the girls' in geometry but not in algebra. Stein obtained conflicting results when using two criteria: "On the basis of the criterion [Cooperative Plane Geometry Test, Form R] boys were significantly better than girls in demonstrative geometry. However, on the basis of teachers' estimates, girls were significantly better than boys."[31] An interesting exception to the superiority of the boys was found by Fadum[32], who stated that girls frequently outscored the boys in ungraded rural districts.

The number of researchers considering reading ability was also six. There was fairly general agreement that this factor was significantly related to success in mathematics. Significantly positive correlations were found between the following:

1. reading ability and generalization achievement in Grade eight arithmetic, Ebert[33]
2. reading grade level and the functional competence of 210 Grade eleven girls, Pitts[34]
3. reading comprehension and Grade eleven geometry achievement, Stein[35]

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[30] Jim Schunert, op. cit.

[31] Harry L. Stein, op. cit., p. 165.

[32] Oscar Fadum, op. cit.

[33] Reuben Ebert, op. cit.

[34] Raymond J. Pitts, op. cit.

[35] Harry L. Stein, op. cit.

4. reading skills and arithmetic problem-solving ability in Grade seven, Treacy[36]
5. reading (including the technical vocabulary of algebra) and algebra problem-solving ability, Kellar[37]

In a more comprehensive analysis of reading ability, Eagle[38] sought to determine the effect of six reading skills; namely, speed, comprehension, general vocabulary, mathematics vocabulary, ability to interpret algebraic formulas. He decided that all were positively (though not all significantly) related to success in mathematics. Of these six, the highest correlation was shown by the ability to interpret algebraic formulas, the lowest by reading speed.

A number of investigators examined progress in mathematics in the light of some form of past achievement. Two factors, arithmetic problem-solving and arithmetic computation, were positively related to achievement in geometry, as shown by Stein[39], and to algebraic problem-solving ability, as shown by Kellar[40]. Challman[41] established

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[36] John P. Treacy, "The Relationship of Reading Skills to the Ability to Solve Arithmetic Problems," Journal of Educational Research, 38:86-95, October, 1944.

[37] Wylma Rose Kellar, op. cit.

[38] Edwin Eagle, "The Relationship of Certain Reading Abilities to Success in Mathematics," The Mathematics Teacher, 41:175-79, April, 1948.

[39] Harry L. Stein, op. cit.

[40] Wylma Rose Kellar, op. cit.

[41] Mildred Challman, op. cit.

ability in the performance of arithmetic and algebra processes as a factor associated with final achievement in geometry. Davis' experiment[42], however, indicated that marks obtained in arithmetic were limited in value as predictors of accomplishment in plane geometry. Guiler maintained that "initial status in computational ability has a marked effect on progress in computation."[43]

Student attitude could conceivably influence achievement. Dislike of mathematics may not be as widespread as is popularly believed, however; for, according to Downey[44], 84% of 6,978 pupils in 15 high schools expressed liking for this subject as against 14% who expressed dislike. In any case, Hamza's experiment[45] indicated that both an unfavorable attitude toward mathematics and an unhealthy attitude toward the teacher were factors of retardation in mathematics. A similar experiment in the field of commercial arithmetic conducted by Billig[46] supported Hamza's findings. In studying the relationship between interest and achievement in mathematics, Torrance concluded that "wherever there are wide discrepancies between aptitude and

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[42] Robert A. Davis and Marguerite Henrick, op. cit.

[43] W. S. Guiler and Vernon Edwards, op. cit.

[44] W. F. Downey, "Mathematical Interests of High School Pupils," The Reorganization of Mathematics in Secondary Education (The Mathematical Association of America, Inc., 1923), pp. 517-23.

[45] Mukhtar Hamza, op. cit.

[46] Albert L. Billig, "Student Attitude As a Factor in the Mastery of Commercial Arithmetic," The Mathematics Teacher, 37:170-72, April, 1944.

achievement in many cases the interest factor seemed to be an important one -- the measure which seemed to explain the discrepancy."[47] Closely associated with these three studies is a survey conducted by Poffenberger[48], which indicated that the following three factors apparently affected attitude toward achievement: (1) parental expectation of achievement, (2) parental encouragement regarding mathematics, and (3) parents' own attitude toward this subject.

Stein[49] found no significant differences in the study habits of average and poor achievers in geometry. Hamza[50], on the other hand, maintained that the differences between the working habits of a retarded group and those of a normal group in mathematics brought about significant differences in achievement. Analyzing the habits of the retarded group still further, he concluded that the following contributed to retardation:

1. attempt to speed up work excessively
2. general disregard for details
3. careless reading and writing and incorrect copying of numbers
4. inaccuracy of drawings and measurements

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[47] Paul Torrance, "Mathematics Interest -- Fundamental or Not?" The Mathematics Teacher, 39:24-26, January, 1946.

[48] Thomas Poffenberger and Donald D. Norton, "Factors Determining Attitudes Toward Arithmetic and Mathematics," The Arithmetic Teacher, 3:113-16, April, 1956.

[49] Harry L. Stein, op. cit.

[50] Mukhtar Hamza, op. cit.

5. use of inefficient and uneconomical methods
6. interference of parents in mathematics homework

Using a sample of 100 Minnesota schools, Schunert[51] showed that the extent of pupil-leadership was not significantly associated with mathematical achievement.

Pitts found a negative but non-significant correlation between the chronological ages of students and their scores on the Davis Test of Functional Competence. This negative correlation "is to be understood in terms of retardation."[52]

Mumma's experiment, in which he compared achievement in English, mathematics, science, and social studies, gave evidence that "in so far as the tests used measure achievement, there is no significant difference in the average achievement of day and resident students in the school subjects investigated in this particular school."[53]

Factors Resident in the Teacher

Method

The most obvious, and possibly the most important, factor to look for in the teacher is the method of instruction. This is borne

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[51] Jim Schunert, op. cit.

[52] Raymond J. Pitts, op. cit., p. 488.

[53] Richard A. Mumma, "A Comparison of the Achievement of Day and Resident Pupils in a Private Secondary School," Journal of Educational Research, 44:106, October, 1950.

out by the large number of researchers who have shown interest in this factor. Out of the 48 studies located, 23 centered around the teacher; out of these 23 again, 20 dealt with the teacher's method. The problem now is to glean all the useful results possible from these 20 experiments.

Homework was the most frequently investigated factor within this group. Lack of agreement characterizes the five experiments examining the question as to whether homework is a factor in mathematical achievement. One researcher, Hines[54], gave an unqualified "yes." Two others, Foran[55] and Anderson[56], answered in the affirmative also, but with one or more qualifications. Schunert[57] maintained that there was no significant relationship between this factor and accomplishments in either geometry or algebra, while Hamza[58] advanced "homework and its consequences" as being significantly correlated with retardation in mathematics. This unusual divergence of opinion would indicate the possibility of numerous other uncontrolled factors enter-

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[54] Vynce A. Hines, "Homework and Achievement in Plane Geometry," The Mathematics Teacher, 50:27-29, January, 1957.

[55] T. G. Foran and Sister Magdalen M. Weber, "An Experimental Study of the Relation of Homework to Achievement in Arithmetic," The Mathematics Teacher, 32:212-14, May, 1939.

[56] William E. Anderson, "An Attempt Through the Use of Experimental Techniques to Determine the Effect of Home Assignments Upon Scholastic Success," Journal of Educational Research, 40:141-43, October, 1946.

[57] Jim Schunert, op. cit.

[58] Mukhtar Hamza, op. cit.

ing into and affecting the results, such as the attitudes of the students and of the parents, and the skill and personality of the teacher. It is interesting to note here that, while Schunert[59] concluded that homework and achievement were not significantly related, his experiment seemed to establish that supervised study for 20 to 30 minutes per day had a significant effect upon attainment.

Four experimenters tried to ascertain the effect of departing from the usual classroom method to a type of laboratory method, in which the students were given practical problems and experiences. In using this method with physically, mentally, and emotionally handicapped pupils, Archer stated that "the laboratory method was found to be more effective in teaching these pupils than the traditional method."[60] Working with two equated groups of normal students, Bishop[61] discovered no significant difference between the groups in so far as the acquisition of knowledge and skills was concerned but established that the experimental group was significantly superior in the application of geometric principles to life situations. Lyda's experiment[62] indicated that the provision of direct and practical

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[59] Jim Schunert, op. cit.

[60] Allene Archer, "Teaching Plane Geometry to Mentally, Physically and Emotionally Handicapped Pupils," The Mathematics Teacher, 44:185, March, 1951.

[61] Albert L. Bishop, An Investigation to Determine the Relative Merits of Two Methods of Teaching Intuitive Geometry -- the Traditional and the Laboratory-field Method, M. Ed. thesis, University of Saskatchewan, Saskatoon, May, 1949.

[62] W. J. Lyda, "Direct, Practical Experiences in Mathematics and Success in Solving Realistic Verbal 'Reasoning' Problems in Arithmetic," The Mathematics Teacher, 40:166-67, April, 1947.

mathematical experiences is a potent factor in problem-solving success. Willits' conclusions[63] were essentially the same as those of Bishop and Lyda, but he added that an emphasis upon an understanding of the situation beyond its "pure skill aspects" would result in more meaningful learning than is possible under the traditional method of teaching.

Three other researchers, Guiler[64], Schultz[65], and Thompson[66], resorted to a more personalized approach, using a considerable portion of teaching time for individualized instruction based on diagnosed weaknesses. All used equated groups, and all agreed that the experimental group gained significantly more than the control. Schultz also observed important personal results in the form of cooperativeness, appreciativeness, and general attitude toward mathematics as by-products of this method. Closely related to the results of these three studies is Hamza's conclusion[67] that one factor in retardation was the failure to provide for individual differences.

The following studies revealed the need to keep mathematics vital

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[63] William M. Willits, "New Objectives for Ninth Grade Mathematics: An Exposition and Appraisal," Journal of Experimental Education, 13:31-45, September, 1945.

[64] W. S. Guiler and Vernon Edwards, op. cit.

[65] Margaret Schultz and M. M. Ohlsen, "A Comparison of Traditional Teaching and Personalized Teaching in Ninth Grade Algebra, The Mathematics Teacher, 42:91-96, February, 1949.

[66] R. B. Thompson, "Diagnosis and Remedial Instruction in Mathematics," School Science and Mathematics, 41:125-28, February, 1941.

[67] Mukhtar Hamza, op. cit.

and interesting. Three pedagogical factors of retardation given by Hamza's study[68] were teaching by rules and over-emphasis of mechanical aspects, stressing speed too much, and trying to develop mathematics as a "state without a foreign policy." The use of regular life application showed a significant positive correlation with success in mathematics, according to Schunert[69]. Two other investigators tried to determine the need and effect of motivation in conducting mathematics classes. The weight of evidence in Shaw's use of goal sheets[70] indicated that goal sheets did further success. Sawin, in an attempt to measure some aspects of motivation through a self-constructed inventory, obtained a significant relationship to achievement as measured by Glennon's Test of Basic Mathematical Understandings but not to the type of achievement measured by the Stanford Achievement Test or by term grades in school. Two reasons were advanced by the investigator to account for this apparent discrepancy in the results. He indicated that the two tests measured different types of achievement. And he said that his motivational inventory "did not provide a measure of _all_ aspects of motivation which affect the manner in which a student applies himself to mathematical studies."[71]

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[68] Mukhtar Hamza, _op. cit._

[69] Jim Schunert, _op. cit._

[70] Robert B. Shaw, "An Experiment in the Use of Goal Sheets in Ninth Grade Mathematics," _Journal of Educational Research_, 37:209-11, November, 1943.

[71] E. I. Sawin, "Motivation in Mathematics: Its Theoretical Basis, Measurement, and Relationships with Other Factors," _The Mathematics Teacher_, 42:91-96, February, 1949.

Three studies centered around attempts to improve the reading of algebraic material. Using a control group, Clark decided that "a higher coefficient of correlation between reading comprehension and algebraic achievement was brought about by means of remedial instruction in the former."[72] Johnson[73] determined that instruction in mathematical vocabulary brought about significant gains in solving problems based on the vocabulary taught as well as in specific learnings, i.e., those learnings measured by the vocabulary and problem tests prepared by the investigator. Holmes[74], on the other hand, detected no reliable difference between experimental and control groups in so far as the solution of verbal problems was concerned.

The use of some specific techniques was examined by Schunert[75]. From his data, he advanced three factors as significantly contributing to achievement; namely, review of subject matter more than once a month, failure rate (2% being better than 10% or more), and the testing of geometry classes more than once each week. Two factors not related were frequency of tests in algebra and the use of

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[72] Mildred I. Clark, "The Effect of Remedial Work in Reading Comprehension upon Algebraic Achievement," The Mathematics Teacher, 32:67, February, 1939.

[73] H. C. Johnson, "The Effect of Instruction in Mathematics Vocabulary upon Problem Solving in Arithmetic," Journal of Educational Research, 38:97-110, October, 1944.

[74] F. M. Holmes, An Experimental Comparison: Algebraic Reading Practice and the Solving of Additional Verbal Problems in Tenth Grade Algebra, M. Ed. thesis, University of Saskatchewan, Saskatoon, May, 1940.

[75] Jim Schunert, op. cit.

supplementary material and of workbooks in geometry.

While working with superior students, Albers[76] noticed that providing enrichment material to an experimental group increased their mathematics-science interest but did not detract from their achievement in the regular algebra work.

Michael compared the effectiveness of the inductive and deductive methods in teaching signed numbers. He concluded that "the outcome of the study [the superiority of the deductive method] seemed to constitute somewhat of a challenge to the contention of modern education theorists for inductive teaching procedures, as far as the teaching of the topics of positive and negative numbers is concerned."[77]

Personality

Cogan[78], as indicated already under key studies, showed that teacher behavior affected student productivity, behavior tending to make the pupils the focus of classroom experiences being superior to the other two types examined. Clarke[79] found that the teacher's attitude (which included his aims and his methods) was one factor

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[76] Mary E. Albers and May V. Seagoe, "Enrichment for Superior Students in Algebra Classes," Journal of Educational Research, 40: 481-95, March, 1947.

[77] R. E. Michael, "The Relative Effectiveness of Two Methods of Teaching Certain Topics in Ninth Grade Algebra," The Mathematics Teacher, 42:87, February, 1949.

[78] Morris L. Cogan, "Theory and Design of a Study of Teacher-Pupil Interaction," Harvard Educational Review, 26:315-42, Fall, 1956.

[79] S. C. T. Clarke, op. cit.

accounting for the ability to reason obtained as a product of teaching geometry.

Preparation

A teacher's T-score (which takes both teaching experience and training in higher mathematics into consideration) showed positive correlation with a pupil's functional competence in mathematics, Alkire[80] concluded. A similar investigation by Schunert[81] showed that teachers with more than eight years of experience obtained best results in algebra, but not necessarily in geometry. According to his results, neither the type of college attended nor the amount of mathematical training received by the teacher made any difference. There was one exception, however; graduates of state universities and of private colleges obtained better results than graduates of teachers' colleges.

The conclusions with respect to teachers' attitudes and training should be regarded in the light of a survey conducted by the Educational Testing Service:

> In a survey of 211 prospective elementary teachers, 150 reported "a long-standing hatred of arithmetic." . . . On the basis of such evidence, says E. T. S., "it seems pretty clear that many elementary school teachers have a hard time keeping even half a jump ahead of their pupils. Their salvation lies in memorized answers, rather than in any genuine understanding of arithmetical concepts. . . . High school teachers . . . fall in this category also.[82]

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[80] G. D. Alkire, op. cit.

[81] Jim Schunert, op. cit.

[82] "Least Popular Subject," *Time*, June 18, 1956, p. 56.

Factors Resident in the Subject

Are there factors in mathematics as a subject which would assist or hinder the pupil in his mastery of its facts, procedures, and so forth? Available research dealt with two possible aspects affecting achievement; viz., organization and content of the curriculum,and components of mathematical ability.

Five researchers dealt with the former. According to Fadum[83], pupils do not achieve equally well in the different types of arithmetical problems. The artificial division of the subject into branches was revealed as a factor of retardation by Hamza's study[84]. In attempting to find means to develop critical thinking, Lewis[85] completely re-organized the curriculum for an experimental group around topics such as the need of clear definitions, the need of assumptions, direct and indirect proof, induction, and data interpretation. His conclusion: whereas the experimental group showed significantly greater gains in critical thinking ability than the control, both groups showed the same progress in the learning of geometry. An experiment was designed by Furst[86] to determine the

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[83] Oscar Fadum, op. cit.

[84] Mukhtar Hamza, op. cit.

[85] Harry Lewis, "An Experiment in Developing Critical Thinking Through the Teaching of Demonstrative Geometry," The Mathematics Teacher, 43:411-13, December, 1950.

[86] E. J. Furst, "Effect of the Organization of Learning Experiences upon the Organization of Learning Outcomes," Journal of Experimental Education, 18:215-28, March, 1950.

effect of the organization of learning experiences upon the organization of learning outcomes. His experimental group was given a curriculum organized in broad fields, was referred to original works instead of prescribed text-books, and was taught by a method emphasizing critical thinking and development of abilities and skills other than recall of information. The results of the study indicated that a clear-cut relationship between the pattern of emphases taught and the pattern learned existed only in the physical sciences and mathematics. The type of mathematics course developed by Willits (discussed in Chapter IV)[87] was not a drawback to those students desiring to continue in mathematics; and it did have definite advantages over the traditional type of curriculum.

In a discussion of components of mathematical ability, one must realize that one is in the field of mental abilities -- a vast field, in which much research has been done. It is not within the scope of this thesis to conduct an intensive survey of this area. It is intended merely to give the conclusions from a few of these studies since the treatment of the thesis problem would be incomplete without any reference to this aspect.

Studies conducted by Blackwell[88], Johnson[89], Murray[90], and

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87 William M. Willits, op. cit.

88 A. M. Blackwell, "A Comprehensive Investigation into the Factors Involved in Mathematical Ability of Boys and Girls," British Journal of Educational Research, 10:143-53,212-22, June and November, 1940.

89 J. T. Johnson, "On the Nature of Problem-solving in Arithmetic," Journal of Educational Research, 43:110-15, October, 1949.

90 John E. Murray, "An Analysis of Geometric Ability," Journal of Educational Psychology, 40:118-24, February, 1949.

Watters[91] gave forth the following as components of mathematical ability:

1. verbal factor -- Blackwell, Johnson, Murray, Watters
2. ability in spatial relations -- Blackwell, Murray
3. reasoning -- Blackwell, Johnson, Murray
4. precision and exactness -- Blackwell
5. number factor -- Murray
6. speed and level of difficulty factor -- Watters
7. geometry achievement factor -- Watters
8. algebra achievement factor -- Watters

It is interesting to note that, according to Murray's data, reasoning made a relatively low contribution to mathematical achievement and that, in Johnson's results, reasoning ranked below vocabulary. The latter concluded:

> It was somewhat surprising to find vocabulary rating highest in both problem-solving and general intelligence. Perhaps it can best be explained by knowing that words are our vehicle of thought and the better the vehicle the better the thinking. And the better thinking one does the more intelligent he is and perhaps the better problem-solver.[92]

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[91] L. J. Watters, Factors in Achievement in Mathematics (Washington: Catholic University of America Press, 1954).

[92] J. T. Johnson, *op. cit.*, p. 115.

CHAPTER VI

SUMMARY OF PRIMARY INVESTIGATIONS

Tabulations of Results and Tests

Each of the tables in this chapter is intended to summarize some aspects of the studies used in the survey, and to focus attention upon prominent tendencies in experimental studies. The tables also indicate some of the inadequacies of past experimentation.

Table III

Table III affords a quick glimpse of the most important conclusions brought out by the investigations. The reader should realize the limitations of such a table. Excessive abridgment inevitably leads to loss of a certain amount of clarity and to possible misinterpretations. Certainly it is impossible to indicate all the ramifications of both factors and results. Caution in reading and interpreting are, therefore, highly recommended. If further information is needed or desired, one may refer to Chapters IV and V of this survey or to the fuller report listed in the bibliography.

In addition to factors and results, the table lists the number of pupils involved in each study, the grade used, and the duration of the experiment. As indicated by the blanks, these items were not always available from reports published in periodicals.

TABLE III

NUMBER OF SUBJECTS, GRADE, DURATION OF EXPERIMENT, AND RELATIONSHIP OF FACTORS TO MATHEMATICAL ACHIEVEMENT

Ref. No.[o]	N	Grade	Duration	Factor	Result of the Experiment
1[*]	64	9	80 days	Providing enrichment material for superior students	Increase in interest in mathematics
2	417	---	---	Sex	Boys superior
				Elementary arithmetic taken in rural rather than in urban schools	Significant superiority of the rural schools
				Number of years of mathematics taken by pupil	Superiority of group having more than 2 years
				Size of school	Schools enrolling more than 500 superior
				Economic status of community	Districts with assessed value in upper fourth superior
				T-score of the teacher	Teachers with T-score in upper fourth superior
3	58	8	---	Effect of homework assignments	An aid if properly assigned and evaluated
5	108	10	1 year	Effect of student attitude	Significant association to achievement
6	80	9	8 weeks	Relative merits of traditional and laboratory-field methods	Lab method superior in teaching how to apply geometric principles

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[o] Number as found in the bibliography

[*] Primary studies discussed in Chapter IV

TABLE III (continued)

NUMBER OF SUBJECTS, GRADE, DURATION OF EXPERIMENT, AND RELATIONSHIP OF FACTORS TO MATHEMATICAL ACHIEVEMENT

Ref. No.	N	Grade	Duration	Factor	Result of the Experiment
8	107	10	2 years	Retention of arithmetic and algebra	Positive correlation to achievement in geometry
9	---	9	8 months	Effect of remedial work in reading comprehension	Significant effect on achievement in algebra
11[*]	1,016	8	---	Intelligence	Significant correlation with achievement in arithmetic
				Type of school: urban, rural, graded rural, ungraded rural	Ungraded rural significantly inferior to others
12	987	8	---	Correlation between achievement	
				and: 1. Inclusive behavior	Significant, positive
				2. Preclusive behavior	Non-significant, negative
				3. Conjunctive behavior	Significant, positive
14	319	9	---	Reading abilities	Positive, non-significant correlation with success
17[*]	400	8	---	Effect of social and economic background	Improvement in both needed for achievement increase

TABLE III (continued)

NUMBER OF SUBJECTS, GRADE, DURATION OF EXPERIMENT, AND RELATIONSHIP OF FACTORS TO MATHEMATICAL ACHIEVEMENT

Ref. No.	N	Grade	Duration	Factor	Result of the Experiment
18	292	7, 8	23 weeks	Effect of remedial instruction based on diagnosed weaknesses	Significantly superior gains made by experimental group
				Effect of increasing length of class period	No significant increase produced
23	32	---	180 days	Correlation between homework and achievement in geometry	Significant difference found favoring home-study group
24*	99	10	5 weeks	Effect of reading instruction on the ability to solve verbal problems in algebra	No significant differences in gains between control and experimental groups
25	598	7	14 weeks	Effect of teaching vocabulary upon arithmetic problem-solving ability	Significant increase in gains
26	241	8	---	Correlation between vocabulary, reasoning and problem-solving ability	Vocabulary and reasoning shown to be factors of problem-solving ability

TABLE III (continued)

NUMBER OF SUBJECTS, GRADE, DURATION OF EXPERIMENT, AND RELATIONSHIP OF FACTORS TO MATHEMATICAL ACHIEVEMENT

Ref. No.	N	Grade	Duration	Factor	Result of the Experiment
27	284	---	---	Intelligence Reading ability Memory Arithmetic computation ability Arithmetic problem-solving ability Knowledge of algebraic vocabulary Fundamental operations of computation in algebra Analysis of verbal problems in algebra	All but memory and arithmetic computation shown to be significantly related to algebra problem-solving ability
28	500	---	1 year	Sex	Few significant differences in achievement
30	---	7	---	The giving of direct, practical experiences	Significantly related to success in solving problems
31	148	8	---	Effectiveness of one-room rural as compared to graded town schools	Town schools significantly superior in achievement

TABLE III (continued)

NUMBER OF SUBJECTS, GRADE, DURATION OF EXPERIMENT, AND RELATIONSHIP OF FACTORS TO MATHEMATICAL ACHIEVEMENT

Ref. No.	N	Grade	Duration	Factor	Result of the Experiment
32	---	9	---	Comparison of inductive and deductive methods	No significant differences found
33	94	10, 11, 12	6 months	Day students versus resident students	No significant differences in achievement
35	210	11	---	Reading grade levels	Significant correlation to functional competence
				Age	No significant correlation
				Gamma I. Q.	No significant correlation
37	635	10	---	I. Q.	Moderate relationship to ability to master algebra
39	116	9	---	Motivation	Significant relationship to mathematical understandings as measured by Glennon Test
40	50	9	2 years	Effect of individual instruction based on diagnosis	No statistically significant differences between groups
41*	100 schools	---	---	See Chapter IV	See Chapter IV

TABLE III (continued)

NUMBER OF SUBJECTS, GRADE, DURATION OF EXPERIMENT, AND RELATIONSHIP OF FACTORS TO MATHEMATICAL ACHIEVEMENT

Ref. No.	N	Grade	Duration	Factor	Result of the Experiment
42	48	9	4 months	Effectiveness of goal sheets	Significant gains made
44	56	7	20 weeks	Remedial instruction based on diagnosis	Significant gains made by experimental group over control
45	500	10, 11, 12	---	Interest in mathematics	Discrepancies between aptitude and achievement often accounted for by interest in mathematics
46	160	7	---	Reading skills	Good achievers significantly superior to poor achievers in all reading skills
48	76	9	1 year	Re-organized curriculum based on new objectives	Significant loss in acquired skills accompanied by significant gain in problem-solving ability

Not all studies appear in the table. Some have been omitted intentionally. All investigations of the mental abilities required for success in mathematics were passed over because this review does not pretend to deal adequately with this aspect of achievement. Some studies were left out because they were concerned with a different emphasis on achievement and would have required lengthy treatment to avoid misinterpretation.

Tables IV and V

Tables IV and V indicate the tests which were used in measuring intelligence and achievement respectively, together with the frequency of use.

Observations from the Tabulations

Table III reveals two significant trends in experimentation. The large majority of studies were concerned with grades from the junior high school. One wonders why. A substantial majority worked with a relatively small sample. Only nine of the 29 studies which indicated sample size used a group of 400 pupils or more.

Table IV shows a heavy emphasis on the tests devised by Otis. As will be explained more fully later, the type and content of intelligence tests may have considerable bearing on the results. If this is true, the great variety of tests, as indicated by the table, would necessitate a closer examination of some of the conclusions.

Two important trends appear from Table V: the large number of

TABLE IV

FREQUENCIES FOR TESTS USED TO MEASURE INTELLIGENCE

Test	Frequency
Otis Self-Administering Test of Mental Ability	7
Otis Quick-Scoring Mental Ability Test	7
Henmon-Nelson Test of Mental Ability	2
Kuhlman-Anderson Test	2
New California Test of Mental Maturity	2
Terman-McNemar Test of Mental Ability	2
American Council of Education Psychological Exam	1
Chicago Primary Ability Battery	1
Dominion Group Intelligence Test	1
Laycock Test of Mental Ability	1
Otis Group Intelligence Test	1
Simplex Group Intelligence Test	1
Stanford Binet Test	1

TABLE V

FREQUENCIES FOR TESTS USED TO MEASURE ACHIEVEMENT

Test	Frequency
Test constructed by the Investigator	17
Averages taken from Term Tests	7
Co-operative Achievement Test in Plane Geometry	3
Co-operative Algebra Test	3
New Stanford Achievement Test	3
Columbia Research Bureau Algebra Test	2
Davis Test of Functional Competence in Mathematics	2
Iowa Every Pupil Test of Basic Arithmetic Skills	2
Analytical Scales of Attainment	1
California Achievement Test	1
Chicago Survey Test in Arithmetic	1
Georgia Algebra Placement Test	1
Glennon Test of Basic Mathematical Understandings	1
Iowa Geometry Aptitude Test	1
Iowa High School Content Examination	1
Sones-Harry High School Achievement Test	1

investigators relying on self-constructed tests, and the wide diversity of standardized tests being used. What effects have these two trends had upon results? Do all these tests measure the same kind of achievement, and with equal reliability? Only a thorough analysis could answer these questions satisfactorily.

CHAPTER VII

SUMMARY OF SIGNIFICANT RELATED STUDIES

A few studies concerned with factors not reported on thus far were located outside of the delimitations of the problem. A number of others added useful information or support to the primary studies dealt with above. A brief reference to some of the pertinent conclusions from these studies will be made in this chapter.

Diagnoses of Difficulties

On the assumption that a teacher's anticipation of students' difficulties would be a factor in improving achievement, Boyd[1] made a diagnostic study of difficulties in first year college mathematics. Her study revealed six major deficiencies:

1. inadequacies in reading meaningfully
2. lack of skill in algebraic and arithmetic manipulation
3. failure to understand fundamental concepts and principles
4. insufficiencies in the ability to see relationships and to generalize
5. limitations in resourcefulness, e.g., various methods of attack
6. faulty memory

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[1] Elizabeth N. Boyd, A Diagnostic Study of Students' Difficulties in General Mathematics in First Year College Work (New York: Bureau of Publications, Teachers' College, 1940).

Memory, she maintained, "is a factor entering into the ability to answer any question."[2] Ohlsen[3] tried to measure the control of fundamental mathematical concepts and skills (as defined by the National Report of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics) by high school students. A self-constructed test of 107 items designed to test the control of 29 concepts and skills was administered to 2,893 students in Grades 10, 11, and 12. Results showed that, on the average, only 48.2% of the pupils knew how to get the correct answer. This percentage, presumably, indicates a general lack of mastery of the concepts and skills fundamental to further progress.

Personality of the Student

Experimenting with 123 adult males, Cattell[4] investigated the association between student personality traits and mathematical ability by correlating ratings on 35 personality trait clusters with scores on the Graduate Record Examination. After intelligence was partialled out, the following traits showed highest positive correla-

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[2] Ibid., p. 56.

[3] Merle M. Ohlsen, "Control of Fundamental Mathematical Skills and Concepts by High School Students," The Mathematics Teacher, 39:365-71, December, 1946.

[4] Raymond B. Cattell, "Personality Traits Associated with Abilities: II. With Verbal and Mathematical Abilities," Journal of Educational Psychology, 36:475-86, April, 1945.

tions:

1. intelligent, analytical (0.33)
2. intellectual, wide interests (0.23)
3. strong-willed, conscientious (0.21)
4. wise, mature (0.20)
5. smart, assertive (0.17)

The traits showing highest negative correlations were as follows:

1. demoralized, autistic (-0.16)
2. changeable, frivolous (-0.14)
3. changeable, characterless (-0.14)
4. character neuroses, psychopathic (-0.14)
5. general emotionality (-0.13)

The last trait raises a vital question. Are some students hampered in their achievement by emotional reactions? Two teachers, Gough[5] and Tulock[6], judging from years of experience, concluded that they were. The latter stated, "Fifteen years of dealing with emotional blocks in mathematics have the author wondering about the classic remark 'there is nothing to fear save fear itself.'"[7] A preliminary study conducted by Malone[8] appeared to confirm the exist-

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[5] Sister Mary Fides Gough, "Mathematophobia: Causes and Treatment," The Clearing House, 28:290-94, January, 1954.

[6] Mary K. Tulock, "Emotional Blocks in Mathematics," The Mathematics Teacher, 50:572-76, December, 1957.

[7] Ibid., p. 573.

[8] William H. Malone and E. L. Freel, "A Preliminary Study of the Group Attitudes of Junior and Senior High School Students towards Mathematics," Journal of Educational Research, 47:599-608, April, 1954.

ence of emotional problems regarding mathematics. A Questionnaire of 130 questions administered to 143 students showed that:

1. Attitudes were charged regarding the practical value of mathematics.
2. Attitudes were charged regarding the "learning environment as socially conditioned."
3. Students desired a more thorough presentation of mathematics.
4. These attitudes were enduring.

An analysis of the effects of students' perceptions of various aspects of the school situation was made by Malpass[9]. Correlations indicated that the way in which students view the school situation was related to achievement as measured by terminal grades but not to achievement as measured by standardized achievement tests.

Community Factors

Shaw[10] agreed with Edmiston (see Chapter V, p. 80) that a significant relationship existed between socio-economic status and academic achievement. Working with social status alone, Ames[11] concluded that the ability to succeed socially was not related to scholastic achievement.

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[9] Leslie F. Malpass, "Some Relationships Between Students' Perceptions of School and their Achievement," Journal of Educational Psychology, 44:475-82, December, 1955.

[10] Duane C. Shaw, "Relationship of Socio-economic Status to Educational Achievement in Grades Four to Eight," Journal of Educational Research, 37:197-201, November, 1943.

[11] Viola Ames, "Factors Related to High School Achievement," Journal of Educational Psychology, 34:229-36, April, 1943.

Of 24 community variables examined by Robert L. Thorndike[12], only two showed correlations with achievement above 0.20. These were the number of professional workers per 1000 of population and the median school grade reached by adults 25 years old and over, with correlation coefficients of 0.25 and 0.21 respectively.

Effect of Teacher-Pupil Interview

Using a random sample of Air Force teachers as interviewers and three different interview techniques, Hoehn concluded that:

> The average teacher, with only a minimal amount of training in interview techniques, cannot improve grade averages by means of interviews. Coupled with the possibility that such interviews may be harmful for some borderline achievers, these results indicate that caution should be exercised in recommending that teachers counsel their students.[13]

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[12] Robert L. Thorndike, "Community Variables as Predictors of Intelligence and Academic Achievement," Journal of Educational Psychology, 43:321-38, October, 1951.

[13] Arthur J. Hoehn and Eli Saltz, "Effect of Teacher-Student Interviews on Classroom Achievement," Journal of Educational Psychology, 47:434, November, 1956.

CHAPTER VIII

CONCLUSIONS

Cautions Regarding Experimental Findings

Having completed the survey, the writer naturally faced the question as to how reliable, in general, the experimental findings of the studies reviewed were. In some cases, one certainly could not accept the stated conclusions without considerable reservation. In many others, the careful reader would not be completely convinced of the validity of the conclusions. Only a very select few of the studies could stand a stringent and thorough cross-examination. Why is this so? The following difficulties imply the chief criticisms which most experimenters are subjecting themselves to when they attempt a study of factors affecting achievement.

Adequate control of intelligence was a major difficulty in many studies. In isolating other factors, the effect of intelligence must certainly be accounted for. Yet not all studies indicated having done so satisfactorily. In any case, the question remains: What is a satisfactory control for intelligence? Equating on the basis of intelligence? If so, which one of the available tests? Do they all measure intelligence equally well? Do they all, as a matter of fact, measure the same "thing"? The following quotation indicates the problem which the researcher may be facing when trying to account for intelligence.

> The first aspect of this view which I must contest is the implicit assumption that intelligence is a single clear-cut mental power or faculty. . . . Thus different tests or test-batteries will inevitably yield somewhat different results, although they have much in common. There is no means of deciding that one test which samples one particular set of abilities is a better test of a hypothetical entity -- intelligence -- than is another which samples a somewhat different set of abilities.[1]

A second difficulty lies in the idea of the "single variable." Good and Scates maintain that this is the main difficulty[2]. The single-variable concept is basic to most attempts at isolating factors of achievement. But has the factor under investigation actually been the only variable? Is it ever possible to keep all other factors constant? The complexity of the problem can be realized partially by a glance at Good and Scates' list of some of the factors involved in the equating of groups -- a list of 27 factors in the pupil, in the teacher, in the school, and in the community[3].

A few specific difficulties could be enumerated also. How much extra zeal does the teacher have in doing an "experiment"? What are the differences in teacher skill and enthusiasm in investigations where two or more teachers are used? What is the effect of the novelty of an experimental setting upon the students? Dodes

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[1] Phillip E. Vernon, "A New Look at Intelligence Testing," Educational Research, 1:3-4, November, 1958.

[2] Carter V. Good and Douglas E. Scates, Methods of Research (New York: Appleton-Century-Croft Inc., 1954), p. 716.

[3] Ibid., p. 708.

lists four other important difficulties involved in performing educational experiments:

> 1. An experiment in education involves relatively few subjects. Unlike a physical or chemical experiment performed upon millions of randomly moving particles, the educational experiment numbering more than a few hundred cases is rather rare.
> 2. A second difficulty derives from the fact that the subjects of these experiments are children, dear to us and important to society. . . . The educator must, however, consider the entire range of changes before recommending any method, device or philosophy.
> 3. A third difficulty arises from the fact that people, unlike molecules, are importantly affected by the experiment per se. . . . The teacher and pupil may react to one or both methods, or to the fact that an experiment is being performed.
> 4. A fourth difficulty is that experimental methods are ordinarily new to the teacher. The effect of practice may alter the results, later on, in either direction.[4]

All these difficulties do not imply the unproductiveness of experimentation in education. Nor does failure to overcome all difficulties in one study necessarily invalidate all the findings. One must, however, be aware of the limitations of each experiment and give them due consideration in forming conclusions. Failure to do so may lead to serious fallacies.

Conclusions as to Principal Factors in Achievement

It would be foolhardy to venture to list the factors according to the order of importance merely on the basis of a survey of studies. To do that, one would need to have a study investigating all factors simultaneously and ascertaining the contribution made by each toward

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[4] Irving Allen Dodes, "The Science of Teaching Mathematics," The Mathematics Teacher, 46:164, March, 1953.

achievement. No such study has been done; hence, the relative importance of factors is still unknown. It is inevitable, however, that a survey of investigations will give some information leading to definite impressions and conclusions.

This review would suggest that the most prominent factors lie in the pupil and in the teacher, probably in that order.

Several important factors resident in the pupil are indicated. The most obvious of these is intelligence; and, while experiments do prove the existence of a significant positive correlation between this factor and mathematical achievement, they also tend to show that this association may not be as great as is generally assumed. A second prominent pupil factor appears to be reading ability. The extent of the mastery of fundamental mathematical terminology and the ability to comprehend clearly and precisely will determine future attainment in mathematics. The Canadian Research Digest, in reviewing the research in reading, concluded:

> Mathematics demands close, precise and detailed reading skills. Algebra and geometry may appear to be far removed from reading, but an analysis of materials and learning difficulties indicates that skill in reading is very much a part of the work in these areas. . . . Much of the reading in both science and mathematics demands attention to detail and is, therefore, slow and meticulous. Rapid reading which is so helpful with literary and social studies material may actually be detrimental for much of the reading in science and mathematics.[5]

A somewhat smaller correlation appears to exist between achievement and a third factor resident in the pupil, viz., the mastery of the

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[5] "Research in Reading -- A Review," Canadian Research Digest, 1:58, Winter, 1959.

theory and skills of previous mathematics courses.

It would appear almost obvious that the method of teaching would affect the achievement of students. The results of many experiments indicate that this is so, but again the magnitude of the correlation may not be as great as is often presumed. Lesser contributions toward achievement appear to be made by the training, the experience, and the attitude of the teacher.

As far as the school is concerned, two factors seem to have a significant effect upon achievement. The size of the school must be such as to afford the necessary facilities for the teaching of mathematics. There also seems to be a definite limit for the size of a class if progress is to remain at the highest possible level.

The evidence for community variables as factors of achievement is largely inconclusive. Ungraded rural schools appear to be at a decided disadvantage. The reason why this is so was not investigated by the researchers dealing with this factor; several, however, indicated that the type and teaching load of the teachers in such schools might account for the poor results.

In conclusion, this survey would suggest that the three most prominent factors affecting achievement in mathematics are the following:

1. the mental ability of the student
2. the reading comprehension of the student
3. the method of presentation used by the teacher

Suggestions for Further Studies

While conducting this survey, the writer became aware of a few important gaps in the investigation of factors affecting achievement in mathematics. Some of these would appear to be extremely important aspects of the teaching-learning situation. If this is so, it is urgent that further research pertaining to these factors be instituted at once.

A few of the areas in which there is a need for further experimentation follow.

1. the effect of the teacher's personality
2. the possibilities in the field of motivation
3. the factors affecting the second approach to a definition of achievement, i.e., the acquisition of general principles, thinking habits, types of reasoning, versatility of attack, etc.
4. the effectiveness of the new emphases in the radically revised curricula being introduced in various states of the U. S. A.

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